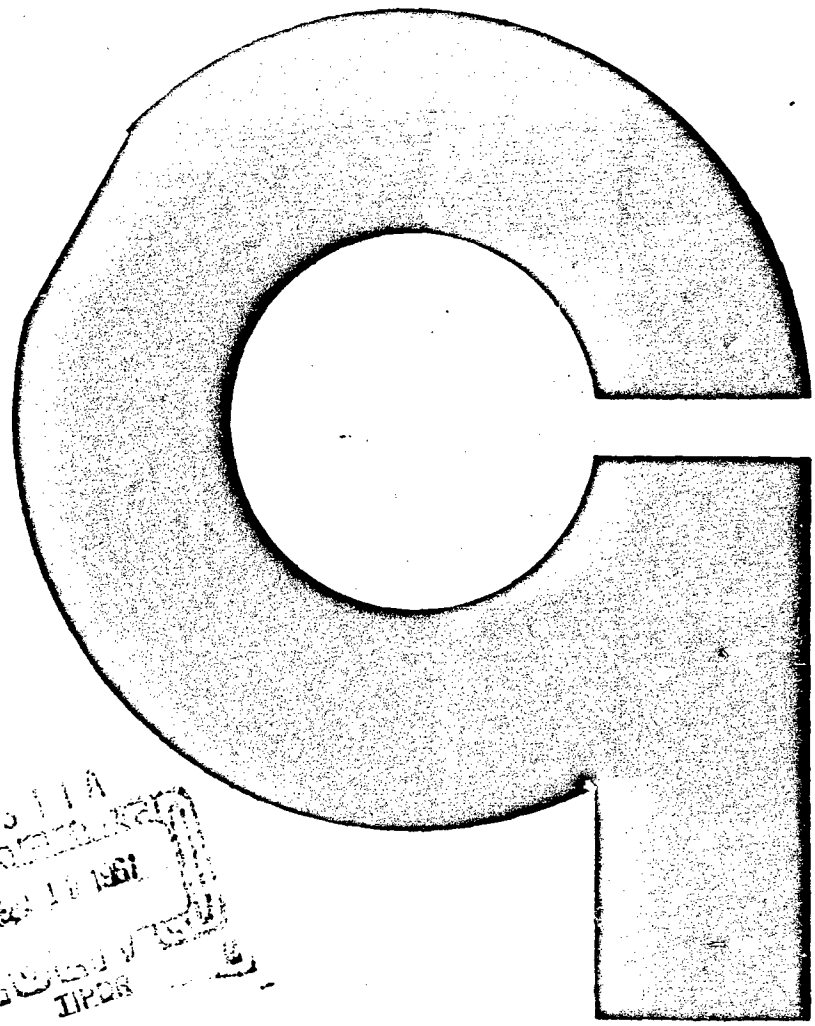

PLASMA REACTION STUDIES

P. J. NAWROCKI R. PAPA

FINAL TECHNICAL REPORT ITEMS I B AND C

CONTRACT NO. AF19(604)-7405

PREPARED FOR ELECTRONICS RESEARCH DIRECTORATE

AIR FORCE CAMBRIDGE RESEARCH LABORATORY

OFFICE OF AEROSPACE RESEARCH

UNITED STATES AIR FORCE, BEDFORD, MASSACHUSETTS

DECEMBER 1961

GEOPHYSICS CORPORATION OF AMERICA BEDFORD, MASSACHUSETTS

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PLASMA REACTION STUDIES

1. Introduction

Air Force Contract 19(604)-7405 was divided into two phases. Phase 1 was to review existing "knowledge" in all aspects of the atmospheric environment and atmospheric physics pertinent to the problem of rf-breakdown. The results of this review (and approximately 90% of the Final Report) were published in book form and distributed among the co-contractors on the breakdown problem. It was anticipated that the review could, if used properly, materially increase the cogency in the experimental programs in rf-breakdown and in the interpretation of these experiments. Accomplishment of this limited objective need not imply that the solution of the problem of predicting rf-breakdown thresholds and the properties of the resulting plasma for all types of terrestrial environments and at all altitudes have been motivated. It does, however, mean that the workers in the field are acquainted with the terrestrial environment as well as all the productive and loss mechanisms of electrons - a state of awareness which apparently did not exist prior to the publication of the review.

Phase 2 was an obvious reaction to Phase 1. In essence, its objective was to list ways and means for filling some of the gaps in existent knowledge concerning both the terrestrial environment (ambient and perturbed) and the many particle-particle and particle-field interactions that in total determine the characteristics of breakdown. More

specifically, the ways and means were to include both laboratory (Phase 2, Part A) and upper atmosphere programs (Phase 2, Part B). For convenience of presentation, both types of recommendations are included in this single document. The order or presentation is selected to follow the scheme of the review and does not imply priority of effort.

As a result of the totality of efforts in the field of rf-breakdown, two important characteristics of the problem were recognized. First, the phenomenological theory of Allis, Brown, etc., becomes less appropriate with decreasing pressure - at least for those cavity experiments of limited size. Moreover, the departure from theory is in a sense such as to over-estimate the breakdwon power. For some configurations, it may be possible to clarify the picture by increasing the dimensions of the cavity, yet this has its inherent drawbacks since there is a corresponding increase in the uncertainty concerning the particular transmission mode of the rf perturbation.

Second, a reliable extrapolation of the cavity experiments (even if the previous objection to the phenomenological theory can be eliminated) must be considered unlikely in view of our relative ignorance of the composition of the upper atmosphere and the roles of the many processes such as solar photo-ionization which can materially alter the breakdown characteristics. The proper theoretical approach would be the more complex Boltzmann equation whose total solution unfortunately remains beyond the present state of the art.

There are ways and means of improving the overall posture of the state of knowledge concerning the interactions of rf energy and slightly ionized gas - extending of course to the phenomenon of breakdown. It is with this general improvement (in the state of the art in mind) that a program is suggested. The following list of items delineates the broad prospective of the program.

(1) Measurements of density, temperature and composition

(2) Measurements of the solar flux, absorption and ionization cross sections

(3) Reaction rates from ion cloud releases and from simple laboratory experiments using photo-ionization and photo-dissociation

(4) Photo and electron impact transition probabilities of molecular species.

(5) Radiation losses in rf-excited plasmas obtained by the spectrometric monitoring of the Bailey, Gordan and Bowles experiments; and the theory of radiation from plasma inhomogeneities

(6) Existing solutions to the Boltzmann equations, the basic postulates involved, and the pertinence of these treatments to the rf-breakdown problem. Where possible, suggestions are made for setting up a tractable simplification of rf-breakdown.

(7) A test of Item 6 can be made by applying the theory to the Stanford experiment of breakdown in a spherical cavity by a remote rf antenna.

(8) A direct experimental attack on the breakdown problem by devising a rocket-borne breakdown experiment.

(9) rf-breakdown in atmospheres perturbed by nuclear detonation and chemical seeding.

In the following sections, each of these topics are discussed in greater detail.

2. The Upper Atmosphere

Of the physical parameters of the upper atmosphere that are of pertinence to the program of rf-breakdown, the most important are density, composition, and temperature. Each of these parameters has been the subject of considerable study; yet, none is in a satisfactory state at the present time - particularly, in regard to the requirements of the rf-breakdown problem.

The review has pointed up many of the variations in density and the sources of these variations. The relation of density to such aspects as the GSP angle (diurnal variation), the geomagnetic latitude (charged corpuscules), and solar activity have been discussed in detail. But most of these characteristics have been established by the integration of measurements over several satellite periods.

On the condition that an upper atmosphere rf-experiment may be performed which requires the local density (and perhaps the density profile) for proper evaluation, this datum would not be available. Furthermore, no reliable data exist concerning the possible derivations from "normal behavior" described in the review. Consequently, we advocate methods by which density may be measured continuously from the ground. In Section 2.2, the advantages of utilizing laser instrumentation as the optical transmitter-receiver are given in detail. As in the case of the present optical density measuring devices, Rayleigh scatter from individual molecular and atomic species provides the basic mechanism.

The utility of the laser device depends upon the pulse width (basically the time required for avalanching of the photons), the power transmitted, and the receiver sensitivity. Pulse width must be small if spatial resolution is to be obtained, and/or scattering from the denser atmosphere eliminated by time-gating. Large transmitting powers, excellent collimation and high sensitivity which are characteristic of the laser are exactly the requirements for a successful density measurement via Rayleigh scatter.

Above balloon heights, the composition is not well known, and therefore the roles of dissociation, mixing, diffusion, etc., have been the subject of considerable conjecture. The problem is inherently more difficult than that of the density, temperature, etc., particularly since trace species may be of considerable importance. If laser instrumentation can be built with sufficient power, sensitivity, and resolution, it may be possible to use this device in composition studies. The basic idea would be to supplement ordinary non-resonant scatter with resonant scatter by the appropriate choice of the frequency. This may mean selecting frequencies in the infrared or near UV, and possibly a fundamental extension of the principle of stimulated emission. The background may also limit the tool to night-time use.

A more immediately available instrument is the mass spectrometer (Herzog version of the Redhead Gauge) presently being used to get the He density profile. The instrument can be used equally well for any specie and has an absolute limit of 10^4 molecules cm^{-3}, or at atmos-

pheric pressure can detect a trace material whose density is 10^{-13} of the total. Of course, the instrument must be rocket or satellite-borne; this is perhaps less convenient and more expensive than ground-based measurements. Details of the system are given in Section 2.2.

2.1 Lasers for Continuous Density, Composition and Temperature Measurements*

In the past, the scattered light from searchlight beams has been used in measurements of upper atmospheric density. The recent development of lasers and masers (sources of highly monochromatic coherent radiation of high total power and extremely high power per cycle per steradian) has made feasible their utilization as geophysical probes. Lasers may be employed in making the following measurements:

(a) Measurements of density and hence temperature as a function of height to 75 kms and possibly higher to 100 kms.

(b) Measurements of height distribution of various atmospheric species via e.g., a laser in the region 2500-3000Å for ozone or an iraser in an appropriate IR region for other species (CO_2).

(c) Possible measurement of winds at extreme altitudes by means of the Doppler shift of the extremely narrow waveband.

(d) Possible measurement of temperature through Doppler shift.

(e) Measurement of dust layers and micro-meteorites.

*Taken in part from J. Pressman (GCA).

2.1 1 Description

The laser as a source has several unique aspects. First of all, light emitted is coherent and this coherence is both latitudinal and longitudinal. So then the laser acts in the manner similar to a radio transmitter. This coherence arises fundamentally out of the nature of the induced emission process. A second major property is that of monochromaticity; this monochromaticity is such that the line emitted has a width much less than the natural width. A third important feature is the very narrow angle of divergence of the beam. The final feature is the large amounts of light power that can be generated.

Amplifiers and oscillators using atomic and molecular processes, as do the various varieties of masers, may extend far beyond the frequency range which has been generated electronically into the infrared, the optical region, or beyond. Such techniques have realized the attractive promise of coherent amplification at these high frequencies and of generation of very monochromatic radiation. As it was attempted to extend maser operation towards very short wavelengths, a number of new aspects and problems arose, which required quantitative reorientation of theory and considerable modification of the experimental techniques used.

In the following discussion, roughly reasonable values of design parameters will be used. For comparison, the characteristics considered in the paper of Schawlow and Townes[2-1] will be presented for masers operating in the normal microwave range. Here an unstable ensemble of atomic or molecular systems is introduced into a cavity which has one

resonant mode near the radiative transitions of these systems. Such an ensemble may be located in a wave guide rather than in a cavity but again there would be typically few modes of propagation allowed by the wave guide in the frequency range of interest. The condition of oscillation of n atomic systems excited with random phase and located in a cavity of appropriate frequency may be written

$$n \geq hV \Delta \nu / (4\pi \mu^2 Q_c) \qquad (2\text{-}1)$$

where n is more precisely the difference $N_1 - n_2$ in the number of systems in the upper and lower states, V is the volume of the cavity, $\Delta \nu$ is the half-width of the atomic resonance at half-maximum intensity, assuming a Lorentzian line shape, μ is the matrix element involved in the transition, and Q_c is the quality factor of the cavity.

The energy emitted by such a maser oscillator is extremely monochromatic, since the energy produced by stimulated emission is very much larger than that due to spontaneous emission or the normal background of thermal radiation. The frequency range over which appreciable energy is distributed is given approximately by

$$\delta\nu = 4\pi kT (\Delta \nu)^2 / P \qquad (2\text{-}2)$$

where $\Delta \nu$ is the half-width at half-maximum of the resonant response of a single atomic system, P is the total power emitted, k is Boltzmann's constant, and T the absolute temperature of the cavity walls and wave guide Since in all maser oscillators at microwave frequencies,

$P >> kT \Delta\nu$, the radiation is largely emitted over a region very much smaller than $\Delta\nu$, or $\delta\nu << \Delta\nu$.

The necessary condition for oscillation may be obtained by requiring that the power produced by stimulated emission is at least as great as that lost due to all loss processes. That is,

$$\left(\frac{\mu' E}{n}\right)^2 \frac{h \nu n}{4\pi \Delta\nu} \geq \frac{E^2}{8\pi} \frac{V}{t} \qquad (2\text{-}3)$$

where μ' is the matrix element for the emissive transition, E^2 is the mean square of the electric field (for a multi-resonant cavity, E^2 may be considered identical in all parts of the cavity), n is the excess number of atoms in the upper state over those in the lower state, V is the volume of the cavity, t is the time constant for the rate of decay of the energy, and $\Delta\nu$ is the half-width of the resonance at half maximum intensity, if a Lorentzian shape is assumed. The decay time t may be written as $2\pi\nu/Q$, but may also be expressed in terms of the reflection coefficient α of the cavity walls.

$$t = 6V / (1-\alpha)\, Ac \qquad (2\text{-}4)$$

where A is the wall area and c the velocity of light. For a cube of dimension L, $t = L / (1-\alpha)\, c$. The condition for oscillation from Eq. (2-3) is then

$$n \geq \frac{\Delta\nu}{\nu} \frac{h(1-\alpha)\, Ac}{16^2 \mu^2} \qquad (2\text{-}5)$$

At low pressure, most infrared or optical transitions will have a width $\Delta\nu$ determined by Doppler effects. Then the resonance half-width

$$\Delta \nu = \frac{\nu}{c} \left(\frac{2kT}{m} \ln 2 \right)^{\frac{1}{2}} \tag{2-6}$$

where m is the molecular mass, k is Boltzmann's constant, and T the temperature. Because of the Gaussian line shape in this case, Eq. (2-5) becomes

$$n \geq \frac{\Delta \nu}{\nu} \frac{h\ (1-\alpha)Ac}{16\pi^2\ \mu^2\ (\pi \ln 2)^{\frac{1}{2}}} \tag{2-7}$$

or

$$n \geq \frac{h(1-\alpha)A}{16\pi^2\ \mu^2} \left(\frac{2kT}{\pi\, m} \right)^{\frac{1}{2}} \tag{2-8}$$

It may be noted that Eq. (2-8) for the number of excited systems required for oscillation is independent of the frequency. Also, this number n is not impractically large. If the cavity is a cube of 1 cm dimension and $\alpha = 0.98$, $\mu = 5 \times 10^{-18}$ esu, $T = 400^{\circ}K$, and $m = 100$ amu, one obtains $n = 5 \times 10^{8}$.

The minimum power which must be supplied in order to maintain n systems in excited states is

$$P = nh\nu/\tau \tag{2-9}$$

This expression is independent of the lifetime of the excited species. However, one must consider that if there are alternate modes of decay of each system, as by collisions or other transitions, the necessary power may be larger than that given by Eq. (2-9) and dependent on details of the system involved. Furthermore, some quantum of higher frequency than that emitted will normally be required to excite the system, which will

increase the power somewhat above the value given by Eq. (2-9). Assuming the case considered above, i.e., a cube of 1 cm dimension with $\alpha = 0.98$, $\lambda = 10^4$ A, and broadening due to Doppler effect, Eq. (2-9) gives $P = 0.8 \times 10^{-3}$ watt. Supply of this much power in a spectral line does not seem to be extremely difficult and obviously has already been accomplished.

The power generated in the coherent oscillation of the maser may be extremely small, if the condition of instability is fulfilled minimally, and hence can be much less than the total power, which would be the order of 10^{-3} watt, radiated spontaneously. However, if the number of excited systems exceeds the critical number appreciably, then the power of stimulated radiation is given roughly by $h\nu$ times the rate at which excited systems are supplied, if the excitation is not lost by some other processes. The electromagnetic field then builds up so that the stimulated emission may be appreciably greater than the total spontaneous emission. For values even slightly above the critical number, the stimulated power is of the order of the power $nh\nu/\tau$ supplied, or hence of the order of one milliwatt under the conditions assumed above.

The most obvious technique for supplying excited atoms is excitation at a higher frequency, as in optical pumping or a three-level maser system The power supplied must be larger than the emitted power in Eq. (2-8). It is not necessary that the pumping frequency be much higher than the frequency emitted, as long as the difference in frequency is much greater than kT/h, which can assure the possibility of negative temperatures. Since, for

the high frequencies required, an incoherent source of pumping power must be used, a desirable operating frequency would be near the point where the most quanta are emitted by a given transition in the discharge or some other source of high effective temperature. This maximum will occur somewhere near the maximum of the blackbody radiation at the effective temperature of such a source, and most often in the visible or ultraviolet region. The number of quanta required per second would probably be about one order of magnitude greater than the number emitted at the oscillating frequency, so that the input power required would be about ten times the output given by Eq. (2-9), or ten milliwatts. This amount of energy in an individual spectroscopic line is, fortunately, obtainable in electrical discharges and this technique has been developed by Javan.

Monochromaticity of a maser is very intimately connected with its amplifier noise properties. If there is considered first a maser cavity for optical or infrared frequencies which supports a single isolated mode, then as in the microwave case, it is capable of detecting one or a few quanta, corresponding to a noise temperature of $h\nu/k$. However, at a wavelength of one micron, this noise temperature is about 14,000°K, and hence not remarkably low. Furthermore, other well-known quantum detectors, such as a photoelectric tube, are capable of detecting a single quantum. At such frequencies, a maser has no great advantage over well-known techniques in detecting small numbers of quanta.

There is now examined the extent to which the normal line width of the emission spectrum of an atomic system will be narrowed by maser action, or

hence how monochromatic the emission from an infrared or optical maser would be. Analysis was made by Schawlow and Townes of the number of excited systems required to produce stimulated power which would be as large as spontaneous emission due to that of a multimode cavity whose frequencies lie within the resonance width of the system. They assumed for the moment that a single mode can be isolated. Spontaneous emission into this mode adds waves of random phase to the electromagnetic oscillations, and hence produces a finite frequency width which may be obtained by analogy with Eq. (2-2) as

$$\Delta \nu_{osc} = (4\pi h\nu/P)\ (\Delta \nu)^2 \qquad (2\text{-}10)$$

where $\Delta \nu$ is the half-width of the resonance at half-maximum intensity, and P the power in the oscillating field. Note that kT, the energy due to thermal agitation, has been replaced by $h\nu$, the energy in one quantum. Usually at these high frequencies, $h\nu \gg kT$, and there is essentially no "thermal" noise. There remains, however, "zero-point fluctuations" which produce random noise through spontaneous emission, or an effective temperature of $h\nu/k$.

For the case considered numerically above, $4\pi h\nu\ \Delta\nu/P$ is near 10^{-6} when P is given by Eq. (2-9), so that $\Delta \nu_{osc} \sim 10^{-6}\ \Delta\nu$. This corresponds to a remarkably monochromatic emission. However, for a multimode cavity, this very monochromatic emission is superimposed on a background of stimulated emission which has width $\Delta\nu$, and which, for the power P assumed, is of intensity equal to that of the stimulated emission. Only if the power is increased by some additional factor of about ten, or if the desired mode

is separated from the large number of undesired ones, would the rather monochromatic radiation stand out clearly against the much wider frequency distribution of spontaneous emission.

The above analysis has been given in some length to indicate the technique of construction of the laser so as to have an adequate concept of the power range, wavelength range and degree of purity of the light emitted. This analysis follows closely Schawlow and Townes' development which is basic to an understanding of the physics of laser operation.

2.1.2 Summary of Existing Laser "Hardware" and Potential Developments

The preceding paragraphs have delineated the physics of the laser device as part of the necessary background for the uses to be discussed later. It is the function of this section to present a broad view of the present state of art in this field as well as its potential development so as to place ourselves in an adequate position to discuss the design of laser geophysical probe devices.

2.1.2.1 Laser "Transmitters"

First, as a preliminary set of remarks, it must be said that the laser field is moving forward extremely rapidly. Steady improvements are being made in power emitted, range of wavelengths available, modulation capability, and total system efficiency. At present, one-half dozen lasers are available as off-the-shelf items. In Table 2-1 are given, for example, the characteristics of "Vireo" lasers, presently commercially available. In addition, many laser components such as crystals, Fabry-Perot end-plate, light sources

TABLE 2-1

CHARACTERISTICS OF VIREO I AND II OPTICAL MASERS

	Beam Width (min of arc)	Direction Errors (min of arc)	Radiated Energy (Watt/sterad Å)	Traverse in Elevation (degree)	Traverse in Azimuth (degrees)	Price ($)
Vireo I	15	±2	2×10^{7}	±15	±90	26,000
Vireo II	1	±0.5	4.5×10^{9}	±45	±360	29,000

for optical pumping, etc., are commercially available for use in the construction of special laser devices designed to meet specific needs.

Present ruby masers have produced pulsed optical beams of 10 kilowatts power or still shorter pulses of perhaps 100 kilowatts peak power. This radiation is concentrated in a band-width of about 0.02 cm^{-1}. Another type of maser, operating continuously at about 0.02 watts, emits a wave which has been shown to be in phase over the entire maser reflector surface and to be concentrated in a frequency interval of about 10 kilocycles. There seems to be no general reason, other than the necessary dissipation of power, why solid state optical masers cannot operate continuously at high power and with a short-time monochromaticity close to theoretical expectation or hence with a frequency width very much less than 1 megacycle/sec.

If a laser produces a wave of wavelength λ with constant phase over a surface of diameter d, the angular width of the radiating beam is approximately λ/d. This can be reduced still further by use of an auxiliary optical system such that the angular width may be reduced to λ/D where D is equal to the diameter and focal length of the lens. The conclusion here is that the angular divergence of the beam is limited only by the optical distortions of the system and not by the nature of the source.

For example, the above 10 kilowatt continuous source would give with a 200" reflector a beam of intensity (according to Townes)

$I = \text{Flux}/(\lambda/D)^2 = 10^{18}$ watts/sterad at 5000 A, or per wave number

$I_\nu = 3\text{x}10^{22}$ watts/sterad cm^{-1} for a bandwidth of $\Delta\nu = 3\text{x}10^{-5}$ cm^{-1}

In addition to the chromium doped ruby, lasers are also being developed using samarium and uranium doped calcium fluoride. These give a variation in wavelength and are more adaptable to CW operation. At present, solid state continuous wave operation is on the verge of actuality.

Continuous wave operation by means of gas discharge has already been achieved by Javan[2-2] at Bell Telephone Labs albeit at relatively low energies. At present, systems are already being designed in the megawatt to 10 megawatt output region by various groups for pulsed operation.

2.1.2.2 Laser Receivers

As mentioned earlier, some conventional devices such as photomultipliers have the capability of detecting a few quanta so that no great advantage is gained here for the use of a laser receiver. However, the major advantage of such a receiver is that it rejects to an extremely large degree extraneous background light and also provides a nearly noiseless receiver. This would hold true even when such a receiver would be aligned with the sun.

Such receivers are not presently on the market and commercially available. However, such receivers, it is expected, will be announced in the next few months. This instrument would be very similar in construction to that of the transmitter. In the receiver, in contradistinction

to the transmitter, the Fabry-Perot end parallel mirrors would be slightly transparent. The major problem here is that the receiver must be exactly matched to the transmitter, i.e., the receiver mirror spacing must be in resonance for the transmitter signal. Additionally, the axis of both receiver and transmitter must be parallel within a very close tolerance to reject extraneous background light.

The internal noise of the receiver, due to its own omni-directional glow, is considered negligible. Also, thermal noise is considered insignificant because the receiver operates at a temperature below that necessary to emit energy in the visible portion of the spectrum. Hence, the laser receiver has a very definite advantage over, say, a photomultiplier in this respect. Such an advantage may be minimized if photomultipliers are used in conjunction with very narrow band filters. But such accessory filters cannot be expected to match the built-in selective natural monochromatic response of the laser receiver.

2.1.3 Scattering of a Searchlight Beam

Because the Rayleigh scatter of a searchlight beam offers a simple but effective method of measuring the density and temperature of the upper atmosphere, it would be desirable if such a method could be extended to higher altitudes (above 60 km) by means of a laser beam. The elements of the scattering technique are presented in the following sections

The light that is reflected by the upper atmosphere into the receiver is proportional to the number of scattering centers present

at the altitude of interest. It is assumed, in the discussion that follows, that the composition of the atmosphere remains constant. Previous results confirm that the method of scattering can be described by the Rayleigh law, in which it is assumed that the particles have diameters that are small compared to the wavelength of the light.

Johnson and Jones have estimated the shape and magnitude of the reflected signal. The discussion that follows, which obtains their result, leans heavily on their presentation.

Data on the upper atmosphere are available from the Rocket Panel, Elterman,[2-3] and others. The magnitude of the pressure and temperature, up to at least 100 km, can be deduced from their results. Hence, it is possible to calculate, using the standard gas laws, the number of molecules per cc as a function of altitude.

Hulbert has given a convenient expression for the amount of light, in lumens, that is scattered back per unit solid angle, per cm length of path, at an angle of ϕ to the incident beam.

$$i_s = \frac{i_o \, 2\pi^2 \, (\mu_\lambda - 1)^2 (1+\cos^2\phi)}{n\lambda^4} \qquad (2\text{-}11)$$

where λ is the wavelength of light, μ_λ the refractive index at wavelength λ, n the number of molecules per cc and i_o the incident beam flux in lumens.

We also have

$$\mu_\lambda - 1 = \alpha_\lambda \, n \qquad (2\text{-}12)$$

with a value of 1.08×10^{-23} for α_λ as given by Johnson, Meyer, Hopkins and Mock.[2-4] Since the variation of α_λ with wavelength is small, its value for these calculations may be taken to be a constant.

The Rayleigh formula is known to be incorrect by a factor of two. If all of the above relations are combined, we find that the intensity of the back scattered light, in lumens, for $\phi=0$, per lumen incident, per cm length of beam, per unit solid angle, is

$$i_s = 9.05 \times 10^{-28} \times n \qquad (2\text{-}13)$$

The intensity of light reaching any particular altitude is equal to the intensity of the light source multiplied by the transmission factor up to the altitude. Similarly, the intensity of the back scattered light, reaching the receiver, is equal to the intensity of the light scattered in the backward direction multiplied by the same transmission factor.

The transmission losses are due to two factors. The first is due to molecular scattering, and applies for all altitudes. The second is due to large particle scattering and applies only to the low altitudes.

The transmission losses that are due to molecular scattering can be computed from the Rayleigh formula

$$\sigma = \frac{32\pi^3 \alpha_\lambda^2 n}{3\lambda^4} \qquad (2\text{-}14)$$

which is obtained from Eq. (2-11) by integration over a sphere. Upon so doing, Johnson and Jones conclude, taking into account losses in the lower atmosphere, that the transmission losses are not great and result

in a total transmission of about 85% for the first 50 km while the losses above 50 km are negligible.

The intensity of the back scattered light entering the receiver searchlight is given by

$$i_s = \text{const } n \times A/h^2 \qquad (2\text{-}15)$$

where A is the area of the receiving mirror and h is the altitude. Utilizing the available data for n, the size of the searchlight, etc., a plot of the expected scattered signal may be obtained. Figure 2-1, taken from the paper of Johnson and Jones,[2-5] shows the estimated signal.

If negligible losses in the searchlight beam above 10 km are assumed, then the ratio of the signal for any two altitudes is

$$n = n_o \, (S/S_o) \, (h/h_o)^2 \qquad (2\text{-}16)$$

where S represents the magnitude of the signal from altitude h at the receiver. If the lower altitude is accessible to the radiosonde, it then becomes possible to compute the density profile, with the radiosonde altitude as the reference altitude.

It is important to note for further reference, that between 15 and 60 km there is a change in the light signal covering four to five orders of magnitude; whereas, from the ground to 60 km the change in the light signal covers 10 orders of magnitude as can be seen from Fig. 2-1. In Fig. 2-2 is given directly the Rayleigh scatter "reflectivity" for back

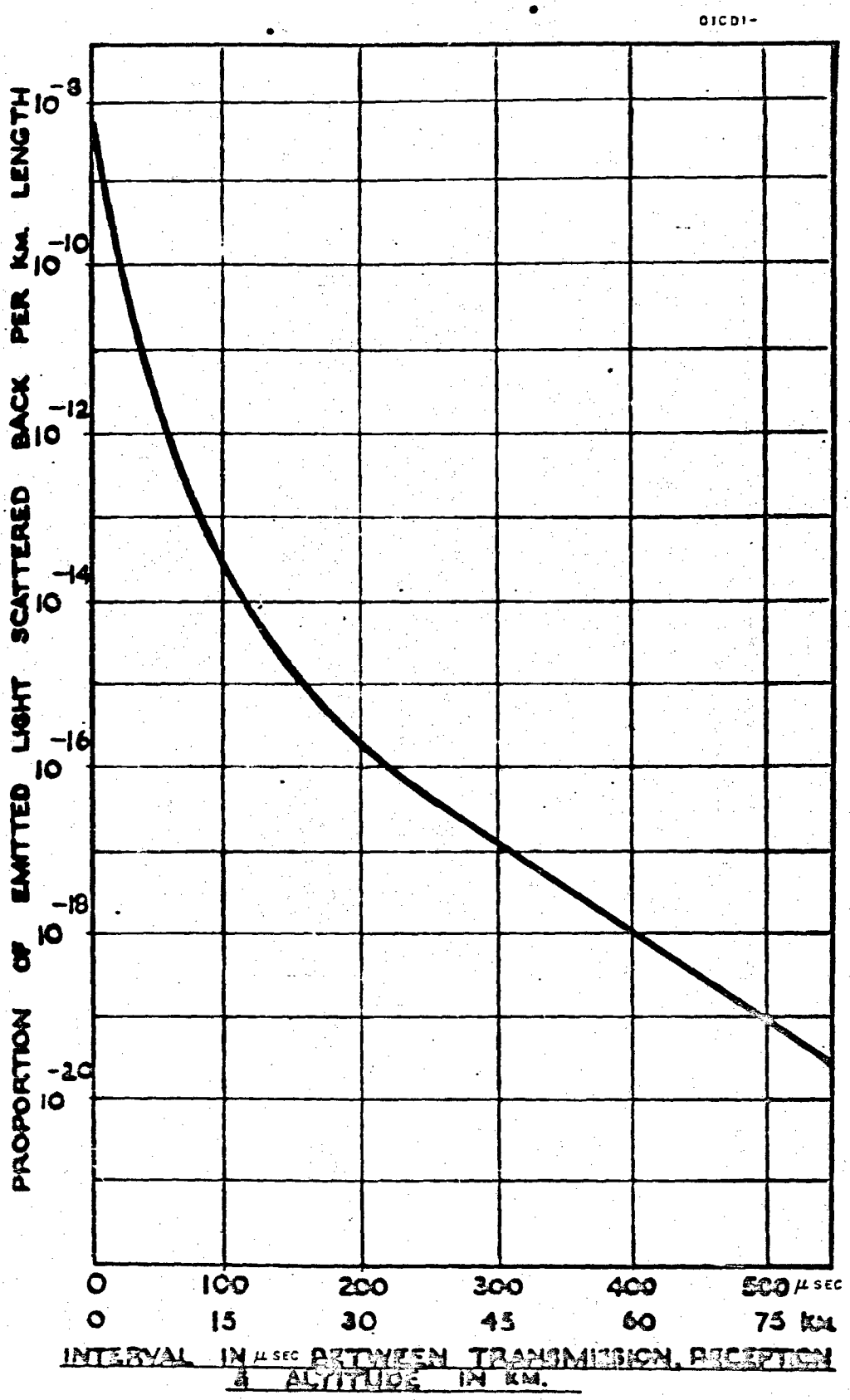


Figure 2-1. Backscattered Light for Johnson Experiment

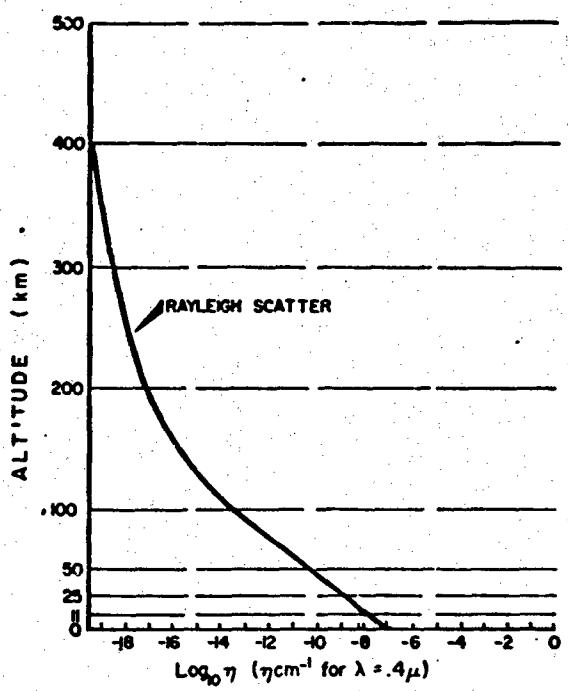


Figure 2-2. Atmospheric Reflectivity

scatter as a function of altitude and in Table 2-2 the data for 4000 and 2000 A are tabulated.

In Fig. 2-3 is given an example of Elterman's results for density and temperature.

2.1.4 Analysis of Laser Possibilities as Geophysical Probe

The purpose of this section is to indicate the geophysical potentialities of the use of lasers in order-of-magnitude fashion. First, there will be indicated its use in measuring densities.

2.1.4.1 Measurement of Densities

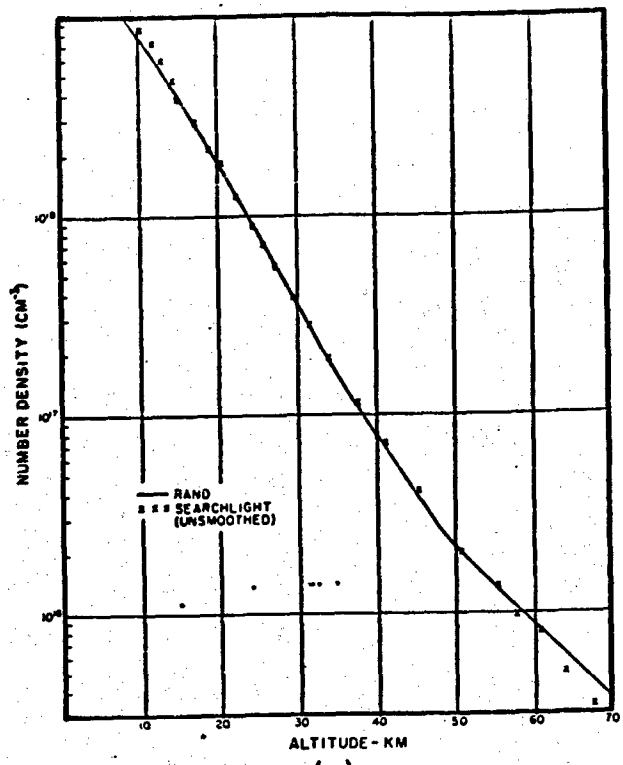
To obtain an indication of the potential use of lasers for this purpose a simple line of argument will be used. Elterman in his experiments used as a source a carbon-arc lamp. The estimated total flux in radiant energy was about 100 watts spread over about 6000 A. The flux for lasers continuously operated can run to about 10 kilowatts. This gives a factor of increase of about 100 in light output. The normal atmospheric density decreases exponentially with a relaxation height of approximately 8 kms for a factor of e. This gives an additional height for probing of about 36 kilometers; so that since Elterman's experiments routinely gave data up to 60 kms, the lasers of the type mentioned should go up to some 36 kms additional or to about 95 kms.

Furthermore, the carbon arc is broad-band, about 6000 A while the laser has a width less than 1 A. This reduction in bandwidth allows the

TABLE 2-2

ATMOSPHERIC REFLECTIVITY (RAYLEIGH)

Altitude kilometers	N cm^{-3}	η cm^{-1} (0.4μ)	η cm^{-1} (0.2μ)
300	1.52×10^{8}	2.87×10^{-19}	4.59×10^{-18}
200	2.95×10^{9}	5.57×10^{-18}	8.92×10^{-17}
100	1.49×10^{13}	2.82×10^{-14}	4.50×10^{-13}
50	2.23×10^{16}	4.22×10^{-11}	6.74×10^{-10}
25	8.32×10^{17}	1.57×10^{-9}	2.53×10^{-8}
11	7.57×10^{18}	1.43×10^{-8}	2.29×10^{-7}
5	1.54×10^{19}	2.91×10^{-8}	4.65×10^{-7}
0	2.55×10^{19}	4.82×10^{-8}	7.70×10^{-7}



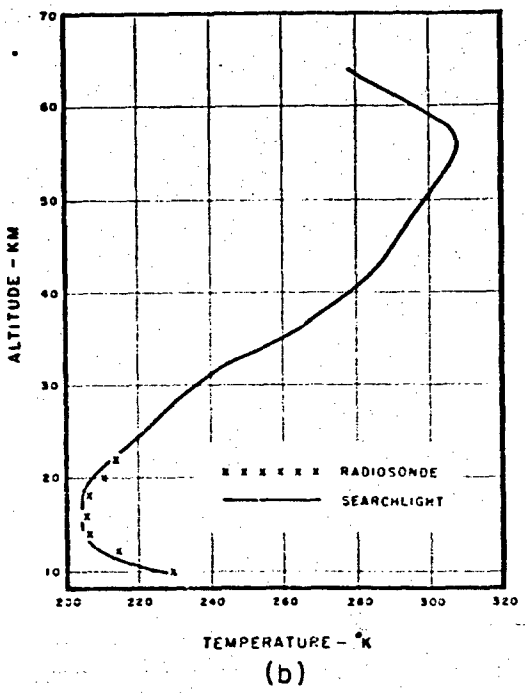


Figure 2-3. (a) Searchlight Density Distribution, New Mexico, 18 October 1952, 1910 to 2210 MST - Elterman Measurement (b) Searchlight Temperature Profile, New Mexico, 18 October 1952, 1910 to 2210 MST - Elterman Measurement

use of a narrowband filter or laser receiver which should cut down the background illumination from the airglow by a factor of 1000. The exceedingly narrow cone of divergence of the laser which is less than 15' of arc gives when compared to Elterman's instrument, which had a divergence of 1.5^{o}, a factor of geometrical narrowing of approximately 100. This should also improve the background noise by a large factor.

The improvement in background by this geometrical narrowing is due to two main factors. First, the natural background due to airglow, zodiacal light, star light, etc., is diminished directly by this narrowing. Secondly, because of the smaller acceptance cone the effect of secondary scattering from the laser light itself is diminished. In the CW case this is a direct effect. In the pulsed case this effect is due to a diminution of tail-off scattering which obscures the returning pulse from the high altitudes.

Also, because of the better height resolution due to the smaller volume anticipated by the transmitter and receptor cones, the measurements of temperatures through the hydrostatic equation can be more exact.

As a check on the analogous argument via Elterman's experiment, another computation is made ab initio. We have

$$F_{\tau} = \frac{\eta \ell A F_{o}}{h^{2}}$$

where

F_{τ} = total flux at receiver

F_{o} = initial flux from transmitter

η = Rayleigh backscatter from a one cm^3 volume

ℓ = length of scattering volume

A = cross section of receiver

h = height of scattering volume

We chose a setup similar to Elterman's. Let

$h = 100$ kms $= 10^7$ cms $F_o = 10$ kilowatts at 3000 A $= 2 \times 10^{22}$ photons/sec

$\ell = 1$ km $= 10^5$ cms, $A \simeq 3 \times 10^4$ cm^2 (area of a two-meter receiving mirror)

and

$\eta_{100} = 3 \times 10^{-14}$.

Hence,

$$F_\tau = \frac{3 \times 10^{-14} \times 10^5 \times 3 \times 10^4 \times 2 \times 10^{22}}{10^{14}} = 10^4 \text{ photons - a detectable amount .}$$

The optimal technique to be used should be investigated. Some of the systems are

1. Continuous emission with crossed beams
   a. steady signal
   b. scatter modulated
   c. rotating beam modulated
2. Pulsed emission
   a. total count of return
   b. envelope analysis

Hence, the second-generation lasers should measure density via Rayleigh scattering up to conservatively 75 kms, and probably higher up to possibly 100 kms

A similar argument holds for pulsed lasers which can run up to 10,000 megalumens (lumen = 1.61×10^{-3} watts at λ of maximum visibility) compared to the 50 megalumens previously used by Friedland et al.[2-6] Also, the pulse length can be much shorter and hence minimizes the tail-off problem that Friedland ran into. Hence, substantial range in altitude probing seems probable over his results.

2.1.4.2 Measurement of Temperature

By use of the hydrostatic equation it is possible to compute the temperature from the density profile in a manner similar to previous attempts. The temperature, as a function of density, is:

$$T = \mu/R \; \rho \int_{w}^{h} g\rho dh$$

where μ = mean molecular weight. The high resolution of the laser device offers the possibility of more exact temperature profiles (due to small angle of divergence).

2.1.4.3 Measurement of Atmospheric Constituents

The measurement of atmospheric constituents by the laser seems a good possibility, particularly in the lower atmosphere, e.g. ozone. Unfortunately, good lasers in the far ultraviolet, < 2500 A are not practical but in the near ultraviolet, ca. 3000 A such lasers may be developed. Also, in the infrared regions many irasers will be available shortly. Because of the infrared absorption of many of the atmospheric gases O_3, CO_2, etc., these irasers should be effective probes. The

vertical distribution of the constituent would be determined from the scattering curve which would not behave in an exponential fashion and hence the deviations would be attributed to absorption. Numerical analyses have been made of this possibility for the carbon arc and will not be repeated here since the computing technique is similar.

The Russians have already made prototype experiments of this kind (without laser) using UV broadband sources.

2.2 The Mass Spectrometer

Herzog et al. of GCA have constructed and flown ruggedized versions of the Redhead mass spectrometer. In essence, the instrument permits the measurement of the profile of a single atmospheric constituent. The sensitivity of the instrument is such as to permit the measurement at atmospheric pressure of an impurity of one part in 10^{11} or at much lower pressures, a density of 10^4 particles/cm^3.

The spectrometer has been operated extensively in the laboratory and its performance has been quite satisfactory. Figure 2-4 shows a helium peak obtained with the system. Figure 2-5 shows the output currents as a function of the pressure in the ion source The slight deviation of the linearity is caused by outgassing and the remote location of the pressure gauge, and will be eliminated in the near future Considerable work has been done to improve the electron multiplier. Electrical breakdown, which occurred frequently in the initial experiments, was eliminated.. It was caused by marginal dimensions of the insulators which have been incorporated in the first flight model.

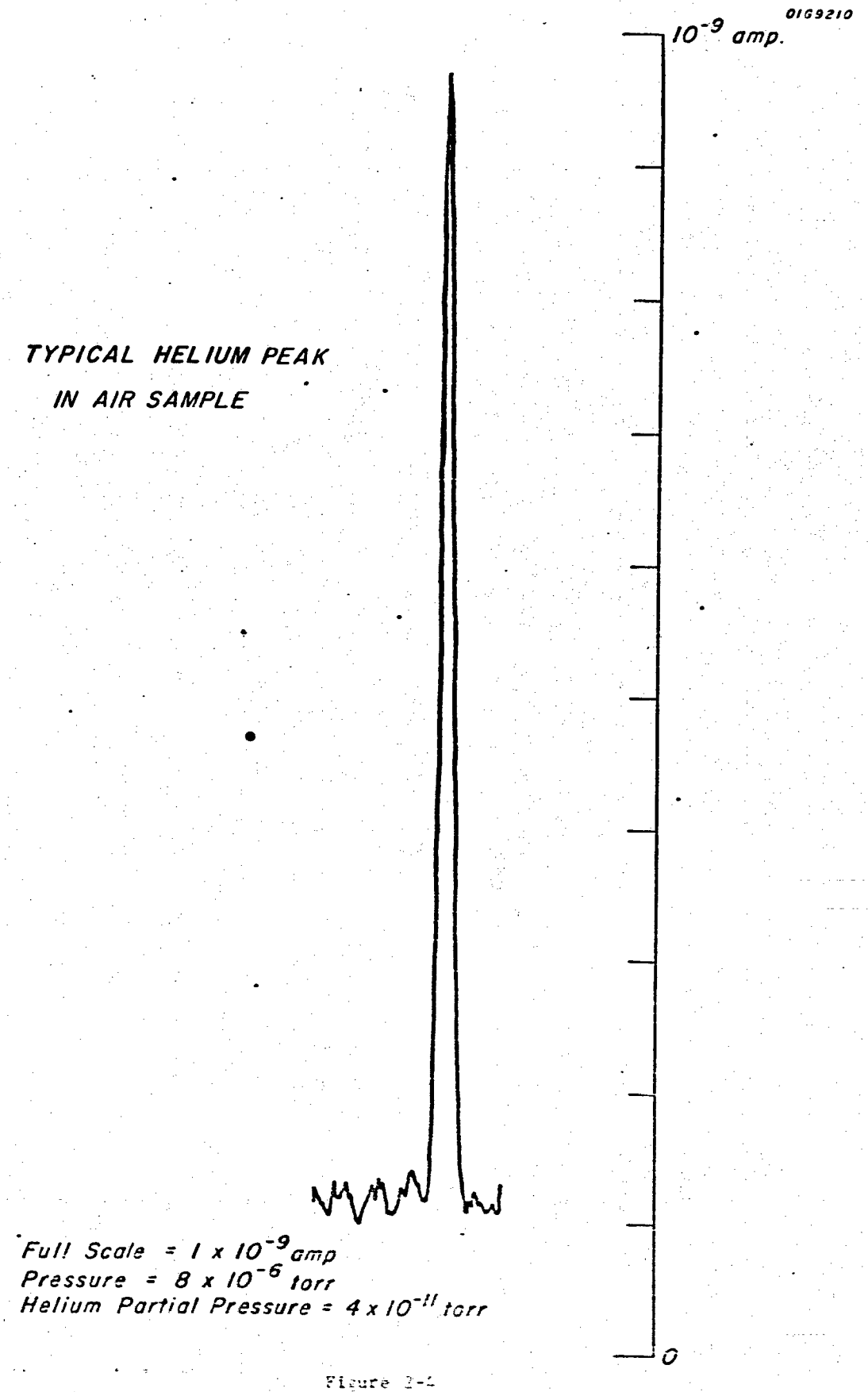

0169210
10^{-9} amp.
TYPICAL HELIUM PEAK
IN AIR SAMPLE
Full Scale = 1×10^{-9} amp
Pressure = 8×10^{-6} torr
Helium Partial Pressure = 4×10^{-11} torr
0

Figure 2-4

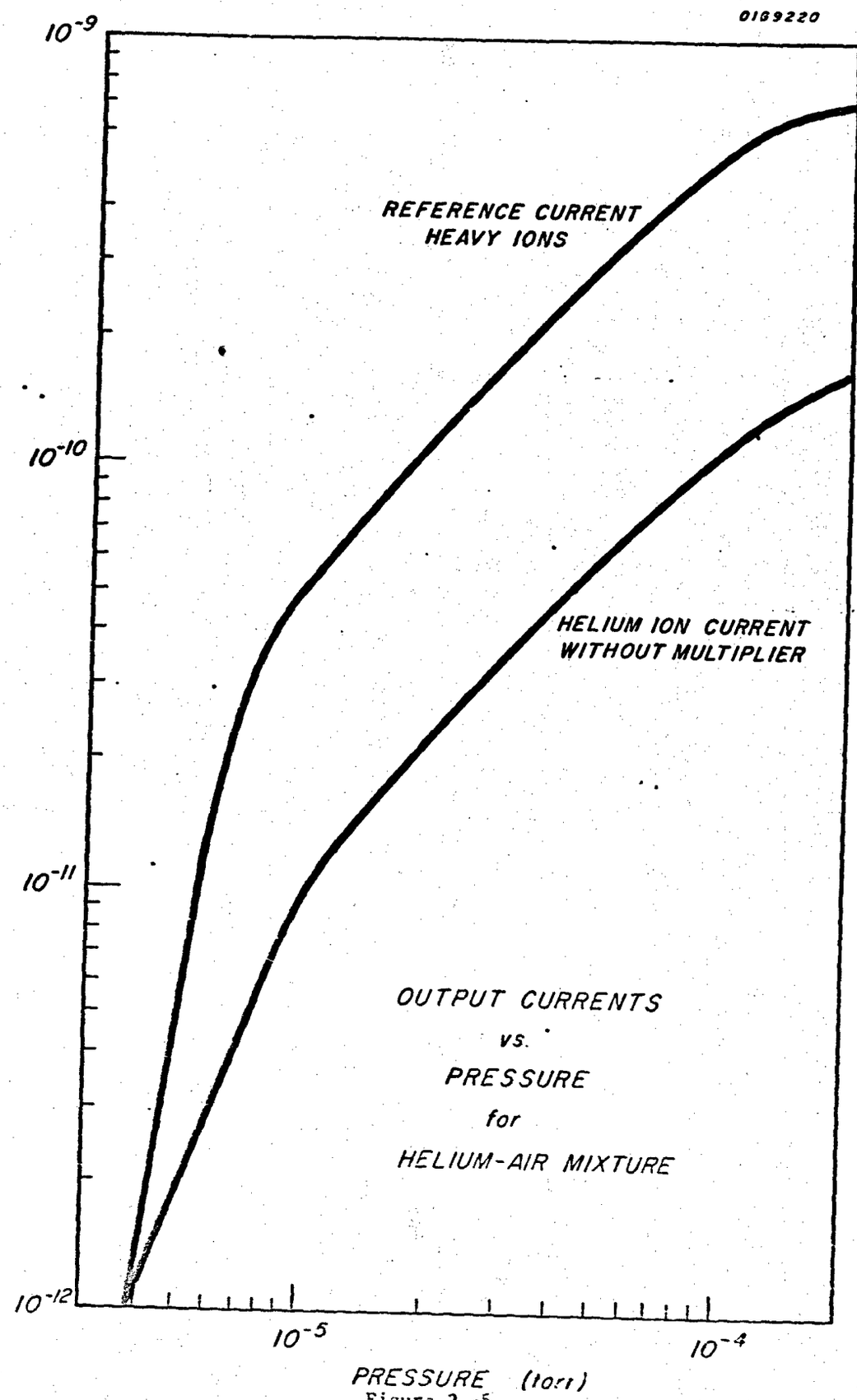

0169220
10^{-9}
10^{-10}
10^{-11}
10^{-12}
REFERENCE CURRENT
HEAVY IONS
HELIUM ION CURRENT
WITHOUT MULTIPLIER
OUTPUT CURRENTS
vs.
PRESSURE
for
HELIUM-AIR MIXTURE
10^{-5}
10^{-4}
PRESSURE (torr)

Figure 2 -5

The electronic equipment associated with the helium mass spectrometer may be conveniently classified into four main categories: the electrometer amplifiers and the three sub-assemblies that comprise the main power supplies, a regulated 26-volt supply, a spectrometer supply, and a thermal regulator system to eliminate voltage drift due to temperature changes. All units have been operated satisfactorily.

Minzner has pointed out that on the basis of an assumed hydrostatic equilibrium, the temperature profile can be obtained from the density profile of a single constituent. Of course, the sensitivity of such a determination would depend upon the scale height (and therefore the mass) of the selected specie.

Total densities could be obtained from a summation of several spectrometers or from the forenamed laser instrumentation.

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2-5 Johnson, E. A. and Jones, RRDE, Report No. 367.

2-6 Friedland, S. S., Katzenstein, J., Sherman, J., Zatzick, M. R., "Improved Instrumentation for Searchlight Probing of the Stratosphere", Physics Dept. Univ. of Conn, Final Research Report (May 1956).

3. Solar Flux, Photo-Ionization and Absorption Cross Sections

The photon flux incident upon a region of the upper atmosphere is determined essentially by the solar flux (outside the earth's atmosphere), the integrated absorption along the path (as determined by composition and photo-absorption cross sections), and the photo-ionization cross section in the region of interest. These parameters then determine the source function for ionization (in the rf-breakdown problem) which is external to the imposed rf-field.

The gross characteristics of line emission and the continuum in the solar spectrum have been delineated in the review. The roles of solar UV and x-rays have been motivated on a semi-quantitative basis. In addition, the photo-ionization and absorption cross sections have been reviewed and listed according to the present state-of-the-art.

There are some notable deficiencies in our information. For example, we do not have information on:

(1) The secular and spatial variations in the solar spectrum outside of the visible.

(2) The complete absorption spectra of the atmospheric species below 1000 A.

These parameters can be obtained. Photon counters and filters for important lines such as monitoring of Lyman-α are available. These should be used on satellites to provide a continuous monitoring of solar radiation.

The absorption work can be done in the laboratory with improved light sources. But the question is, "Are the variations in these parameters critical to the rf-breakdown problem?" The answer at present is that the sophistication of the theory is so limited that these parameters although important are not introduced into the calculations. Consequently, it would appear that they indeed might even be deferred to a later stage in the development of the theory.

4. Reaction Rates

Because of the obvious importance of the field of reaction rates to the behavior of the atmosphere, both ambient and perturbed, the field is being very actively pursued at the present time. The existence of such a broad program will then impose certain ground rules for any suggestions on the improvement of the current state-of-the-art. In short, these are: to refrain from duplicating existing experiments and to avoid expensive and complicated techniques that might require a considerable extension of present capabilities and therefore involve long periods of time before concrete results are obtained. Within this framework, the following two-fold program involving both atmospheric seeding and laboratory instrumentation is suggested.

4.1 Reaction Rates from High Altitude Chemical Releases

Marmo et al[4-1] have indicated that chemical releases in the upper atmosphere under varying conditions such as altitude, degree of solar illumination (using the atmosphere as a controllable screen), specie, etc., can be used effectively to get rates for many of the reactions pertinent to the dynamics of the upper atmosphere.

In a typical chemical release in the altitude regime of interest here (90-120 km) alkali metals such as sodium, cesium, or lithium may be used. They are released at high temperature which, combined with solar radiation and possibly other processes, results in the creation of a small percentage

of free electrons. The chemicals expand initially to approximately the atmospheric density and then continue to expand radially (but less rapidly) due to diffusion processes. During this latter phase of the expansion the electron population is depleted by the various attachment processes. Radar observations of the expanding electron cloud then measure the rate of electron depletion. From this, from the altitude dependence discussed above, from the fact that well-defined chemical (or electron) clouds can be seeded at selected altitudes, and from the fact that a wide selection of suitable chemicals for seeding purposes are at the disposal of the investigator, one should be able to extract reliable values for chemical reaction rates of interest.

In general, the success of this tool depends upon several factors:

(1) The ability to produce and/or reproduce a particular chemical release (e.g., reaction temperature, completeness of reaction, etc.).

(2) The ability to observe significant characteristics of the chemical cloud (electron density by radar, atoms by optical emission, etc.).

(3) The ingenuity of the researcher in the fabrication of models such that the various processes, say, the loss processes of electrons (diffusion, recombination, attachment, etc.) can be readily delineated

These three vital aspects have been achieved and have formed the basis of evaluation and interpretation of both the initial series of releases by Marmo et al[4-2] and the much more inclusive series of releases by

Rosenberg et al[4-3]. It might be pointed out that the conclusion of this series of tests does not mean that there has been full utilization of the electron cloud as a tool for investigating the parameters of the upper atmosphere. There are other uses (of the contaminant release) which have provided the motivation and predominated in the design of the tests.

The phenomenology of the release depends upon such parameters as chemical yield of the contaminant, thermal ionization efficiency, ambipolar and neutral diffusion, photo-ionization cross sections, chemical consumption, etc. Depending upon the pertinence of the interaction to a particular release, the investigator must be able to pull out piecemeal each of these parameters. It would be remiss to enter into a detailed description of each and the appropriate solution of each type of release as a function of altitude. Actually, the dynamics of the release is given by the continuity equation, and the solutions of a score of these characteristic equations were given in Chapter 7 of Atmospheric Processes. However, it is advantageous to demonstrate that each atmospheric parameter can be distilled out if sufficient ingenuity is applied. Consider, for example, the parameter of diffusion. Figure (4-1) shows the application of a particular mathematical model to a series of releases which varied in altitude. Note that the selected mathematical model of diffusion control is completely successful in giving the time history of the release for release altitudes of 110 to 138 km. At 94 and 100 km, other interactions such as chemical consumption and attachment are important enough to materially effect the electron density. But, and this is the

essence of the entire matter, the releases under a diffusion-controlled configuration have made it possible to separate out the diffusion contribution in the complex release. In a similar manner, each and every atmospheric parameter including rates of pertinent chemical reactions can be defined or evaluated.

4.1.1 Radiative Attachment to Atomic and Molecular Oxygen

An examination of a typical reaction is offered as an example of the dynamics of contaminant releases in the upper atmosphere, the choice being attachment to oxygen. The dominant processes of attachment of electrons in the pertinent region of the upper atmosphere are:

two-body radiative attachment

$$O + e \rightarrow O^- + h\nu$$

$$O_2 + e \rightarrow O_2^- + h\nu$$

three- body attachment

$$O_2 + e + O_2 \rightarrow O_2^- + O_2^*$$

Biondi (as well as Bloch and Bradbury at an earlier date) has measured in the laboratory the three-body rate of attachment for molecular oxygen, the resulting rate being of the order of 3×10^{-30} cm^6 sec^{-1}. Similiar laboratory measurements on the rates of radiative attachment in molecular oxygen have not been achieved in the laboratory. One reason, of course, is the relative speed of the competitive process of three-body attachment. In order to circumvent this difficulty, it is necessary to go to much

smaller pressures, but then the radiative flux becomes too small to measure and/or wall effects become important. As for atomic oxygen, measurement on such a specie in the laboratory is not realistic since the lifetime is too small.

How radiative attachment rates of both atomic and molecular oxygen can be accomplished by cogent experiments in the upper atmosphere is sketched as follows.

When a point-release electron cloud is formed the chemicals expand almost immediately to a particle density approximately that of the ambient after which they diffuse outward much more slowly. Man release experiments have indicated that the radial particle distribution in the electron cloud is approximately Gaussian and remains so for a relatively long period of time. The important parameter for this distribution is the Gaussian half-width r_o. The second important parameter is the diffusion velocity which is implicitly contained in the diffusion coefficient D. Both r_o and D have been determined rather accurately for many different altitudes and for many different types of chemical clouds. Above an altitude of about 100-110 km (see Figure 4-1) the chemical cloud expansion is clearly diffusion controlled and chemical consumption and turbulent processes are unimportant. However, below these altitudes the latter two processes do become important, and though complicate the analysis, the contributions of these processes are separable.

The essential characteristics of releases in the exponentially varying

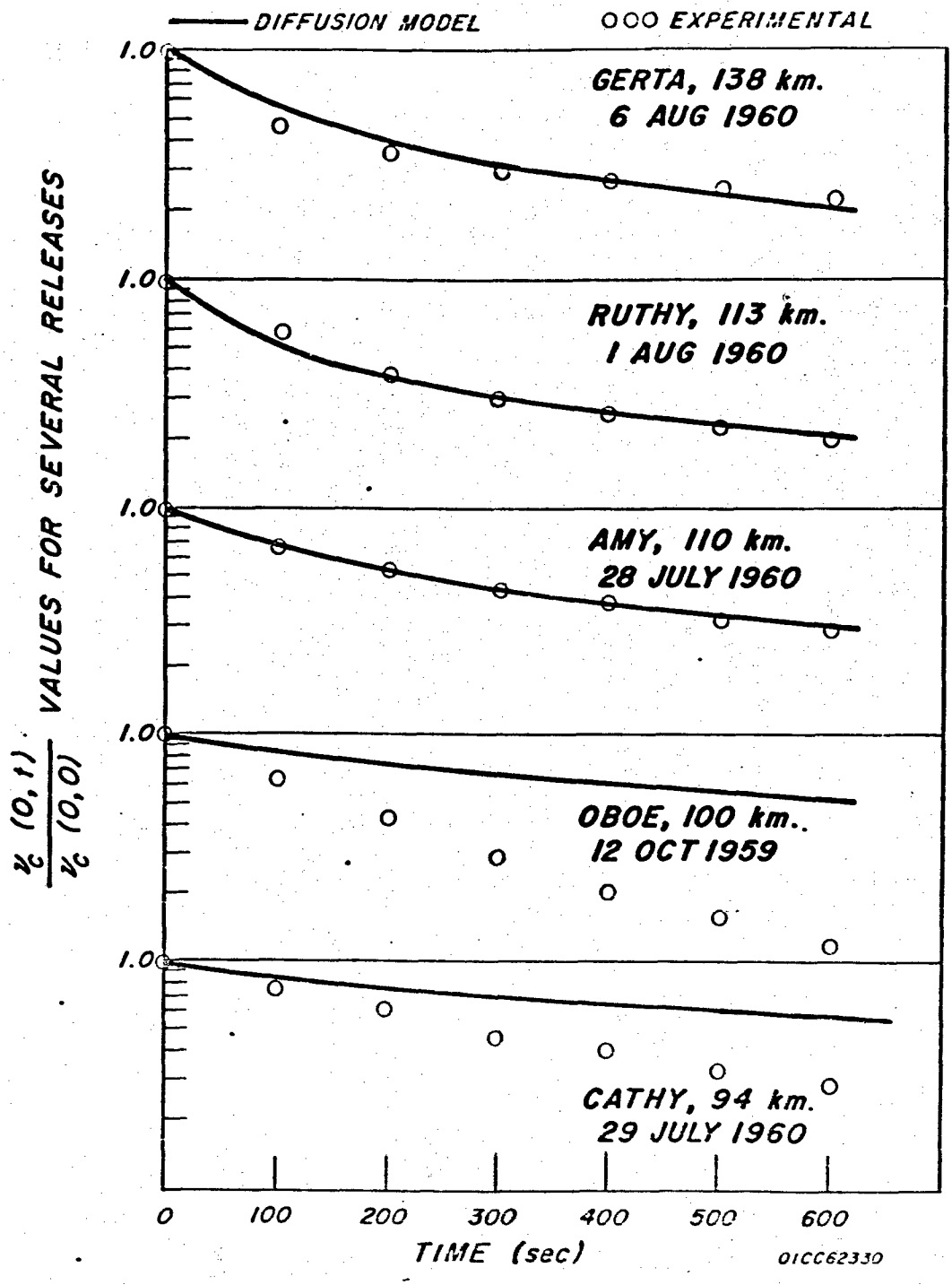
DIFFUSION MODEL
OOO EXPERIMENTAL
1.0
GERTA, 138 km.
6 AUG 1960
1.0
RUTHY, 113 km.
1 AUG 1960
1.0
AMY, 110 km.
28 JULY 1960
1.0
OBOE, 100 km..
12 OCT 1959
1.0
CATHY, 94 km.
29 JULY 1960
0
100
200
300
400
500
600
TIME (sec)
01CC62330
VALUES FOR SEVERAL RELEASES
$\frac{\nu_c(0,t)}{\nu_c(0,0)}$

Figure 4-1

atmospheres are:

(1) In the altitude regime between about 90 km and 120 km the oxygen content in the atmosphere suffers a marked change. At 90 km the concentration of molecular oxygen is about 100 times that of atomic oxygen. As the altitude increases above 90 km the molecular oxygen photo-dissociates so that at about 105 km the concentrations of molecular and atomic oxygen are approximately the same, and at 120 km the concentration of atomic oxygen is about 10 times that of molecular oxygen. These facts are clearly depicted in Figure 4-2 where the atmospheric concentrations of atomic and molecular oxygen are shown as functions of altitude. The importance of this profile is that it enables the investigator to sort out three general altitude regimes: chemical interactions involving molecular oxygen predominate say between 90 and perhaps 100 km, those involving atomic oxygen predominate above 115-120 km, and interactions involving both atomic and molecular oxygen are jointly important for altitudes like 100-110 km.

(2) Atmospheric temperature and pressure both vary considerably between 90 and 120 km. The pressure dependence should provide the means of isolating three-body reactions (these are more important at the higher pressures, hence at the lower altitudes) from the two-body radiative reactions which should be more important at the higher altitudes. To what extent chemical release measurements can be refined so as to yield temperature data remains to be determined.

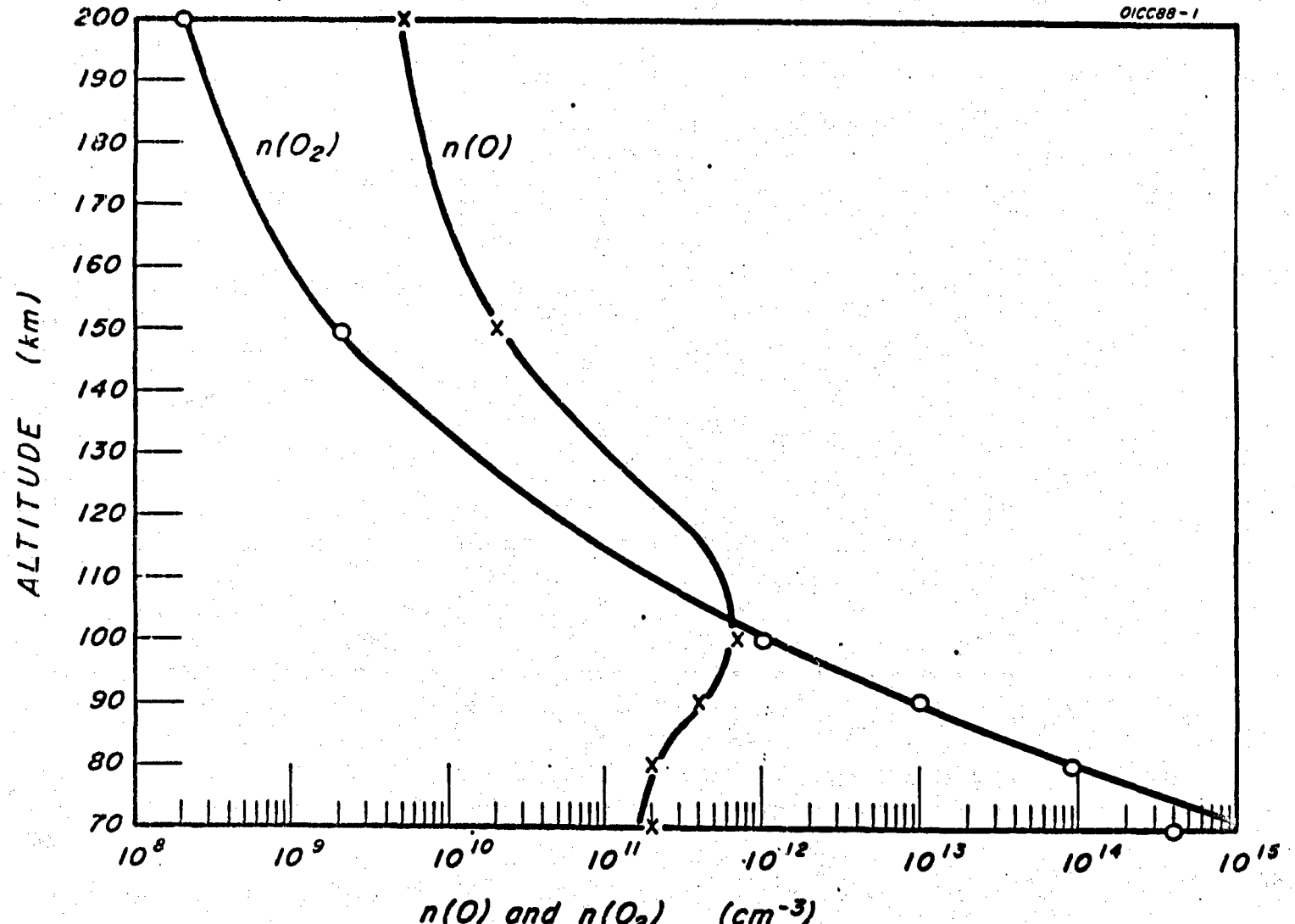

Figure 4-2 Atomic and Molecular Oxygen Concentrations as Functions of Altitude

(3) Both atomic and molecular oxygen have strong positive electron affinities whereas molecular and atomic nitrogen have negative affinities.

(4) Atmospheric constituents other than oxygen and nitrogen are present in only very small amounts.

(5) In the altitude regime between about 90 and 120 km chemical seeding techniques are particularly amenable to chemical reaction rate analysis. This is because chemical consumption is minimal and because diffusion rates are relatively slow thus allowing for long observation times.

Item (1) should provide the means of isolating the rates of radiative attachment to atomic and molecular oxygen. Item (2) should enable one to distinguish two-body radiative attachment from three-body attachment. Item (3) can distinguish reactions involving oxygen from those involving nitrogen. Item (4) extends this distinction to all constituents other than oxygen. And Item (5) extends the analysis to constituents which do not naturally occur in the upper atmosphere yet retains a correct environment free of wall effects.

It is possible to state the case in a sophisticated manner; for measurements at a sufficiently high altitude so that three-body attachment is not important but sufficiently low so that the presence of molecular oxygen is still important the attachment probability is

$$\beta(h) = k_1[O] + k_2[O_2] \qquad (4\text{-}1)$$

where the brackets [ ] indicate concentrations. Then from several measurements of β at different altitudes, and from a knowledge of $[O]$ and $[O_2]$, we can in principle determine k_1 and k_2.

At the lower altitudes where three-body attachment predominates similiar expressions for rate constants of three-body attachment to atomic and molecular oxygen could be obtained.

Collision cross sections σ_i may be simply obtained from the rate constants k_i in case the σ_i are temperature and pressure independent. The following formula is then generally valid:

$$k_i = \sigma_i \, v_{ir} \, e^{D_i/kT} \left(\frac{D_i}{kT} + 1\right) \qquad (4\text{-}2)$$

where D_i is the interaction energy and v_{ir} is the relative velocity.

4.1.2 Associative Detachment

A second example of the scheme is the reaction of associative detachment

$$O^- + O_2 \rightarrow O_3 + e$$

Dalgarno states that for this reaction a rate as high as 10^{-10} cm^3 sec^{-1} cannot be excluded. Yet a simple examination of the data on high altitude chemical releases indicates that a maximum rate for this reaction is 10^{-13} cm^3 sec^{-1}.

For an optimum effort in rates by high altitude releases, new experiments would be conducted, and perhaps even new tools such as the Puerto Rico dish (employing incoherent back scatter of electrons) applied. However, in the modest effort proposed here, the costs of conducting supplementary tests are circumvented.

4.2 Reaction Rates from Laboratory Program

Marmo has suggested a group of experiments to measure some of the present unknown rates employing such selective techniques as photo-dissociation. For example, consider the much discussed reaction of atom-ion exchange, which was originally assigned a rate by Bates as high as 10^{-9} cm^3 sec^{-1} (later estimated as 10^{-12} cm^3 s^{-1})

$$O^+ + N_2 \rightarrow NO^+ + O$$

It has been customary in measuring rates of such reactions to create the ion specie by electron impact. Unfortunately, such a means is not particularly selective, and a number of ionic species and excited states are formed in addition to the one desired. This complicates the measurement considerably. Marmo's idea is to employ specific photo-dissociation and photo-ionization to create a particular specie. In the above measurement, commencing with a mixture of N_2 and O_2, we have:

$$O_2 + h\nu \text{ (1470 or 8.45 ev)} \rightarrow O\ (^1D) + O\ (^3P)$$

$$O\ (^1D) - h\nu \text{ (11.6 ev)} \rightarrow O^- + e$$

$$O^- + N_2 \rightarrow NO^+ + N$$

The ionization and dissociation energies of the gases involved are:

	Ionization (ev)	Dissociation (ev)
$O(^3P)$	13.6	
$O(^1D)$	11.6	
O_2	12.2	5.12
N_2	15.6	9.76
NO	9.$^+$	6.5

The energies of the selected photons are therefore such that formation of NO^+ proceeds by a unique process. The ratio $[NO^+]$ $[O^+]$ as measured in a mass spectrometer can then be related to the rate.

In summary, then, a modest program combining simplifed laboratory measurements with the detailed study of release of contaminants in the upper atmosphere gives definitive results which can improve the existent state-of-the-art for reaction rates.

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5. Transition Probabilities

In the sequence of events leading to breakdown by the coupling of rf energy to a slightly ionized gas, inelastic collisions between electrons and neutrals result in the excitation of electronic, vibrational, and rotational levels. The lifetimes of these levels (essentially the inverse of the transition probability) then determine the energy loss due to radiative transitions. Whether or not the radiated photon escapes from the medium depends upon the population of the lower (energy) state and the photo-transition probability. The relative (oscillator) strengths for the various means of excitation are given roughly by the characteristic figures for electronic excitation, 0.1 to 0.5 ev for vibrational levels, and 10^{-2} to 10^{-4} ev for rotational levels. These figures are to be compared to the average energy gain for electrons between collisions; essentially all the energy is randomized in a single collision, the factor being (1-m/M). In air at an equivalent altitude of 30 km, the breakdown field is of the order of 10^{7} watts/m^2. Dividing by the flow velocity yields the energy density ($\epsilon_o E^2$) which is of the order of 0.04 joules/m^3, or E ~ 400 volts/cm. The mean free path of electrons at this altitude is about 5×10^{-4} cm, so that the energy gain per collision is of the order of 10^{-2} volts. Since this is of the same order of this vibrational energy increment, the importance of the cross sections for these inelastic collisions is clearly demonstrated. It appears that electronic excitation in oxygen occurs at 2.7 ev, but there is also evidence of vibrational excitation at a few tenths of an ev. The manner in which these various factors enter into the Boltzmann equation is given in Section 7.2.

The subject of transition probabilities of all atomic species which are relavent to the ambient atmosphere (an atmospheres contaminated by particular chemical seedings) has been adequately treated by A. Naqvi in Atmospheric Processes. To give an estimate of the accuracies "conferred" upon the values for atomic transitions, it is considered that the theoretical values are good to ± 10% while the experimental uncertainty is ± 30%. In spite of the inability of the experimentalists to substantiate the theoretical results, the latter are considered to be on firm grounds established by the quantum theory.

The question now is - what is to be done to advance the state-of-the-art so as to provide information on transition rates which may be essential to a complete understanding of the mechanics of breakdown. There are three steps which suggest themselves and they are given in the order of increasing complexity as well as degree of importance to the breakdown problem. These are: First, to extend the calculations on atomic species to other heavy atoms. These atoms can result from nuclear detonations but the extension may be more of esthetic than practical value. The second and more important step is to calculate the transition probabilities of the molecular species (e.g., O_2, N_2, NO, etc.). The former can be accomplished by a relatively simple extension of the screening theories of Layzer (Section 5.1). However, the problems introduced by going to molecular species (Section 5.2) are inherently more complicated. The third step is to obtain the cross sections for excitation of these various states by electron impact. Unfortunately, there is no unambiguous procedure of employing the results of the more tractable photo-excitation to the process of excitation by electron impact.

5.1 The Complex Atom*

The probability amplitude for a radiative transition A → B in a many-electron atom is proportional, in a first approximation, to the matrix element $\langle B|\vec{u}\cdot\vec{D}|A\rangle$, where $\vec{D}$ is the total electric moment of the atom and $\vec{u}$ is the polarization vector of the absorbed or emitted photon. If one makes the further approximation of assigning the states A and B to definite configurations, the matrix element reduces to a product of two factors, one involving spin and angular coordinates only, the other involving radial coordinates only. Since all matrix elements connecting a given pair of configurations have the same radial factor, the spin-angular factor, which can be evaluated by means of standard methods in the theory of atomic spectra, determines the relative strengths of Zeeman components in a line, in lines in a multiplet, and of multiplets in a transition array. Thus the determination of absolute transition probabilities hinges on the radial factor.

One can allow for configuration interaction by assigning the states A and B to mixtures of pure configurations. Once the components in such a mixture have been chosen, their relative amplitudes can be evaluated with little difficulty. The transition matrix element $\langle B|\vec{u}\cdot\vec{D}|A\rangle$ may therefore be regarded as a known linear combination of matrix elements connecting pure configurations, and the problem reduces formally to one in which configuration interaction no longer figures explicitly, though in fact, as we shall see, its most important

*From David Layzer (CCA)

effect still remains to be taken into account.

The radial factor is proportional to the transition integral

$$I(n\ell,\ n'\ell') = \int_0^\infty P_{n\ell}\ r\ P_{n'\ell'} dr\ , \qquad (5\text{-}1)$$

where $P_{n\ell}$ and $P_{n'\ell'}$ are the radial wave functions of the active electron in States A and B respectively. The factor of proportionality is a product of integrals of the type

$$I_o(n\ell) = \int_0^\infty P^A_{n\ell}\ P^B_{n\ell}\ dr, \qquad (5\text{-}2)$$

one for each inactive electron, and is normally very close to unity.

Although the radial transition integral depends explicitly on the radial functions of a single electron, each of these depends implicitly on the entire set of radial functions for the state in question. Thus, if configuration interaction is neglected, the best radial functions for a given atomic state satisfy a set of simultaneous integrodifferential equations (the Fock equations), a typical member of which has the form

$$\left[-\frac{1}{2}\frac{d^2}{dr^2} + \frac{\ell\ (\ell+1)}{2r^2} - \frac{Z - S_{n\ell}(r)}{r} \right] P_{n\ell} = E_{n\ell}\ P_{n\ell} + X_{n\ell}. \qquad (5\text{-}3)$$

The screening function $S_{n\ell}$ and the exchange function $X_{n\ell}$ are actually functionals of all the radial functions in the set, and their forms depend on the quantum numbers S and L as well as on the configuration quantum numbers.

Allowing for configuration interaction by assigning the states A and B to specific mixtures of configurations, one arrives by way of the

variation principle at a set of "generalized Fock equations". A typical member of this set has exactly the same form as a typical Fock equation, but the exchange term $X_{n\ell}$ in the generalized equation includes contributions arising from configuration interaction as well as from exchange. This is the effect of configuration interaction.

Numerical solution of the Fock equations, even in their original form, presents too many practical difficulties to serve as an adequate basis for extensive calculations of f-values. Besides, such a procedure would tell very little about the accuracy of the results. One needs to simplify the equations, not only in order to be able to solve them, but also in order to gain insight into how various aspects of their structure affect the accuracy of transition integrals evaluated from their solutions.

Owing to the form of the integrand, the value of a typical transition integral depends strongly on the behavior of the radial functions at large distances from the nucleus. This behavior is in turn largely determined by the energy parameters relating to the active electron in the initial and final states. Finally, the energy parameters are related to ionization energies whose values can be found by experiment. All these relations are of fundamental importance to the problem.

5.1.1 Asymptotic Form of the Radial Functions

As r increases, $S_{n\ell} \rightarrow N-1$, where N is the number of electrons in the system, and $X_{n\ell} \rightarrow 0$, so that the Fock equation for $P_{n\ell}$ reduces to the radial wave equation for an electron in a Coulomb field,

$$\left[-\frac{1}{2}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{2r^2} - \frac{Z - N + 1}{r} \right] P_{n\ell} = E_{n\ell} P_{n\ell} \qquad (5\text{-}4)$$

It then follows that

$$P_{n\ell} \sim A \left[\frac{Z\,(n^* - \ell - 1)!}{n^{*2}\,(n^* + \ell)!} \right]^{1/2} W_{n^*,\ell-1/2}\left(\frac{2Zr}{n}\right) \qquad (5\text{-}5)$$

$$(Z \equiv Z - N + 1)$$

where $W_{n^*,\ell-1/2}$ is Whittaker's hypergeometric function, and n*, the effective principal quantum number, is defined by

$$E_{n\ell} = -\frac{Z^2}{2n^{*2}} \qquad (5\text{-}6)$$

The normalization factor A has been chosen so as to have the value ℓ when n* is an integer. Apart from this factor, the asymptotic form of $P_{n\ell}$ thus depends entirely on the value of $E_{n\ell}$ (or n*).

The parameter $E_{n\ell}$ has a very simple interpretation. Let U^+ denote a given electron configuration minus an $n\ell$ electron, so that $(U) = (U^+)(n\ell)$. When the system as a whole is in a definite energy eigenstate Γ, the subsystem whose configuration is U^+ will not normally be in a definite one of its energy eigenstates but in a mixture of them. Let $E(U^+|\Gamma)$ denote the expectation value of the energy of the subsystem when the system as a whole is in the state Γ. If $E(U|\Gamma)$ denotes the absolute energy of the state Γ, then $E_{n\ell} = E(U|\Gamma) - E(U^+|\Gamma)$. Thus $E_{n\ell}$ is the expectation value of the energy of the system relative to the "core" (defined as the system minus an $n\ell$ electron). This result remains valid if the state Γ is assigned to a mixture of configurations instead of just one.

The energy of the core is not the same as the energy of the ion formed by removing an $n\ell$ electron from the atom. The former is always numerically greater (smaller in magnitude) than the latter, because the $n\ell$ electron tends to shield its companions from the field of the nucleus.

It is convenient to define the energy of a system relative to its parent ion, $E^*(U^+|\Gamma)$, by analogy with the definition of $E(U^+|\Gamma)$. Thus $E^*(U^+|\Gamma)$ is minus the expectation value of the minimum energy needed to ionize the system. Though unconventionally defined, this quantity does admit in principle of direct experimental determination. In practice, only the energies of the ionic states are known, not their probabilities. However, the latter can be calculated by means of standard methods in the theory of complex spectra. We may therefore look upon the energy of a system relative to its parent ion as an observable quantity. Denoting this quantity by $W_{n\ell}$, we have

$$W_{n\ell} = E(U|\Gamma) - E^*(U^+|\Gamma), \tag{5-7}$$

so that

$$W_{n\ell} - E_{n\ell} = E(U^+|\Gamma) - E^*(U^+|\Gamma) > 0. \tag{5-8}$$

This difference represents the effect of screening by the $n\ell$ electron on the energy of the core and need not be small compared with $-W_{n\ell}$. Some recent quantitative studies show that in neutral atoms $(W_{n\ell} - E_{n\ell})$ is appreciable compared with $-W_{n\ell}$ whenever the core contains one or more electrons that belong to the same shell as the screening electron, and increases with the number of such electrons. For the ground state

of O, with its eight L electrons, the contribution of 2p electron to the core energy turns out to be several times as great as the ionization energy itself. These studies also show that the relative importance of the screening contribution diminishes with increasing degree of ionization along an isoelectronic sequence.

When the principal quantum number of the active electron exceeds the principal quantum numbers of all the core electrons, its contribution to the core energy is usually small, though not always negligible.

5.1.2 The Coulomb Approximation

A transition, and the states it connects, is "quasi-hydrogenic" if, in both states, the principal quantum number of the active electron exceeds the principal quantum numbers of all the core electrons. Except in alkali-like atoms, transitions to or from the ground states of many-electron atoms rarely qualify for this label. On the other hand, quasi-hydrogenic transitions between excited states are very common.

Most, though probably not all, quasi-hydrogenic transitions satisfy the following two conditions:

1. The active electron shields the core electrons so ineffectively that $W_{n\ell} \approx E_{n\ell}$ and $W_{n'\ell'} \approx E_{n'\ell'}$.

2. The value of the transition integral depends almost entirely on the contributions from a range (r_c, ∞) within which the radial functions may be placed by their asymptotic forms.

If these conditions are fulfilled and if the energies $W_{n\ell}$ and $W_{n'\ell'}$ are known from experiments, the transition integral can be evaluated, apart from the normalization factors A and A', since it can be approximated by an integral over the range (r_c, ∞), within which the integrand is known to a good approximation. Extensive tables of such integrals have been published by Bates and Damgaard;[5-1] these incorporate earlier calculations of the same kind by Hylleraas.[5-2] The principles underlying the approximation, however, have been exploited much earlier by a number of authors, though in a less explicit and less systematic fashion.

Let $r_{n\ell}$ denote the average nuclear distance of an electron whose radial function is $P_{n\ell}$ ($r_{n\ell} = \int_0^\infty P_{n\ell} r P_{n\ell} dr$), and let ρ denote the geometric mean of $r_{n\ell}$ and $r_{n'\ell'}$. The pure number $1 - Q$ with $Q = I(n\ell, n'\ell')/\rho$, measures the extent of cancellation between positive and negative contributions to the transition integral I. By Schwarz's lemma, $Q < 1$, with equality only when $n\ell = n'\ell'$. The smaller Q is, the more stringent becomes the second of the two conditions given above.

This condition may be written in the form

$$\int_0^{r_c} P_{n\ell} \frac{r}{\rho} P_{n'\ell'} dr \ll Q. \qquad (5\text{-}9)$$

If $Q \approx 1$, this inequality is weaker than the conditions

$$\int_0^{r_c} P_{n\ell}^2 dr \ll 1, \qquad \int_0^{r_c} P_{n'\ell'}^2 dr \ll 1 \qquad (5\text{-}10)$$

for the normalization factors A and A' to be close to 1, so that in some circumstances one may be able to improve the approximation by using more accurate values for A and A' than A = A' = 1. Seaton[5-3] has shown how to derive such values from the energy spectra in which $W_{n\ell}$ and $W_{n'\ell'}$ are imbedded.

Even when the two conditions that are necessary for the validity of the Coulomb approximation are satisfied, the quantum defects n-n* and n'-n'* may be substantial. Only for very highly excited states are the quantum defects negligible. These states are hydrogenic, not merely quasi-hydrogenic. For them, the Coulomb approximation is trivial. Thus the domain within which the approximation can usefully be applied seems to have rather clear-cut boundaries.

5.1.3 Beyond the Coulomb Approximation

Before turning to specific proposals for going beyond the Coulomb approximation, let us consider in a general way what this would involve.

Transitions in which the active electron belongs to the same shell as one or more of the inactive electrons do not normally meet either of the two conditions previously mentioned. If the active electron belongs to the same subshell as one or more of the inactive electrons, it is almost certain that neither is fulfilled. This means that we can no longer rely on experiment to furnish $E_{n\ell}$, and therefore we can at most hope to develop methods that will apply to transitions for which the

energy parameters can be predicted with sufficient accuracy.* Moreover, the effects of the nonconstant part of the screening function $S_{n\ell}$ and of the exchange function $X_{n\ell}$ on the form of the radial functions can no longer be ignored.

Because one can always evaluate energy parameters more simply and with greater precision than wave functions, it is useful to regard the evaluation of $E_{n\ell}$ as the first step toward a determination of $P_{n\ell}$. Given $E_{n\ell}$, together with suitable approximations to $S_{n\ell}$ and $X_{n\ell}$, one could integrate the equation for $P_{n\ell}$ inward from infinity. The solution would deteriorate, of course, as it approached the origin, and ultimately diverge; but such solutions could still yield accurate estimates for transition integrals.

5.1.4 Application of the Screening Theory

As is well known, the predictions of the standard theory of atomic spectra concerning absolute term energies are subject to large errors of a systematic nature, arising from the neglect of configuration interaction. Recently, an alternative theoretical scheme has been proposed for describing complex spectra (Layzer[5-4]), whose principal features are as follows:

1. By allowing the nuclear charge Z to enter explicitly into

*"Sufficient accuracy" means in practice that the effective principal quantum numbers n^* and n'^* must be known to within small fractions of unity. The smaller the value of Q, the more accurately the energy parameters need to be known.

all the calculations, one is able to classify the effects of configuration interaction in a physically significant manner and to work consistently to a given order of approximation.

2. Certain interactions between configurations are taken into account in the first approximation, namely, those connecting states belonging to the same "complex", a complex being defined as the aggregate of all states belonging to a given set of principal quantum numbers and a given parity.

Within the framework of the approximation defined by the requirements that Z enter explicitly into all the calculations and that all interactions connecting terms in the same complex be taken into account, the simplest theory is that based on screened hydrogenic radial functions. The best screening parameters are those defined by the variation principle. In spite of its simplicity, the screening theory accounts quantitatively for the main regularities characterizing term spectra in isoelectronic sequences. Its predictions are also free from the systematic errors associated with the usual theory.

The predicted term energies have the form

$$W = W_2 Z^2 + W_1 Z + W_0 + O(Z^{-1}), \qquad (5\text{-}11)$$

apart from relativistic corrections. The first two coefficients are given exactly by the theory, the remaining ones only approximately. The absolute accuracy of these predictions, so far as it has been tested, appears to be quite good in moderately and highly ionized systems.

In neutral atoms, the terms of order 1/Z are often conspicuous, and the agreement between the screening theory and experiment is less good.

Let us now return to the problem of determining suitable radial functions for evaluating transition integrals. To begin with, we are now in a position to fill a gap that we left earlier in our discussion of the generalized Fock equations. We assumed that the states A and B could each be assigned to definite mixtures of pure configurations. It is now clear that we must allow for all interactions within a given complex. The mixture coefficients, or relative amplitudes of the configurations in a given mixture, are then independent of Z and their values are given by the screening theory. We do not in this way allow for <u>all</u> the effects of configuration interaction, but we allow for the most important ones and we ensure the correctness of the first two terms in the expansions of the $E_{n\ell}$.

In a first approximation one could use values for $E_{n\ell}$ and $E_{n'\ell'}$ calculated according to the screening theory. As for the troublesome terms $S_{n\ell}$ and $X_{n\ell}$, these are of relative order 1/Z compared with the other terms in the equation for $P_{n\ell}$. Probably it would be a sufficiently good approximation to use screened hydrogenic functions to evaluate them. The equation for $P_{n\ell}$ would then reduce to either a single, independent, linear integro-differential equation or to an inhomogeneous differential equation, depending on how one dealt with the occurrences of $P_{n\ell}$ in the functional $X_{n\ell}$. In either case a numerical solution would present no difficulty. The nuclear charge Z would appear explicitly in the equation,

and could probably also be retained in the solution. Thus a single numerical solution would furnish values of transition integrals for homologous transitions in an entire isoelectronic sequence.

5.1.5 Transition Integrals in Highly Ionized Systems

In an approximation that becomes increasingly accurate as Z increases along an isoelectronic sequence, one can evaluate transition integrals by using screened hydrogenic radial functions, the screening parameters being given by the screening theory. Varsavsky[5-5] has carried out a detailed comparison between the results obtained with this method and those obtained by means of more elaborate and (presumably) more accurate methods. For certain classes of transitions (e.g., transitions in which the principal quantum number undergoes no change) he found a remarkably close correspondence between the two sets of predictions, extending even to transitions in neutral atoms.

Since the comparison data were largely of unknown accuracy, the meaning of this result is not entirely clear. However, to some extent one can estimate the accuracy of transition integrals calculated on the screening approximation independently. A given integral can easily be evaluated as an explicit function of Z and of the screening parameters for the initial and final states. This integral is accurate to terms of relative order $1/Z^2$. Now, small changes of the screening parameters will also produce changes of relative order $1/Z$ in the value of the integral. Moreover, one can estimate the inherent uncertainty in the values of the screening parameters. Thus one can estimate <u>one</u> of the contributions to

the error; and it is not unreasonable to suppose that the remaining contributions will usually be of the same order of magnitude. In this way we arrive at a simple and useful criterion: If the value of a transition integral is insensitive to small changes in the screening parameters it is fairly reliable.

5.2 Transition Probabilities of Molecular Species

In Atmospheric Processes, the Morse potential functions for O_2, N_2, and NO as calculated by Forrest Gilmore (Rand Corporation) were presented. It was indicated (e.g., by inclusion of the repulsive state in NO suggested by Marmo) that the listing of the Morse curves were incomplete. Recently, Miescher[5-6] has discovered several more of these repulsive states. These should be explicitly added to the overall picture since their inclusion is essential to the proper interpretation of experiments on lifetime of metastable state, etc. Gilmore also listed approximate values of these lifetimes (inverse of the transition probability) of some of the metastable states involved. Again, these listings are both incomplete and insufficiently well determined. In the interactions of waves and plasmas, in addition to the electronic transition probabilities, one requires the vibrational and rotational transition probabilities.

The theoretical vibration eigenvalues E_ν can be derived from the series:

$$E_\nu = h c \omega_e (\nu + 1/2) - h c \omega_e x_e (\nu + 1/2)^2 + h c \omega_e y_e (\nu + 1/2)^3 - \ldots \qquad (5\text{-}12)$$

which essentially determines the shape of the potential well.

Usually ω_e, $\omega_e x_e$ and $\omega_e y_e$ are experimentally determined. The vibrational wave functions are, of course, interrelated to the vibrational eigenvalues and the assumed shape of the potential function, that of the Morse function being

$$V(r) = D_e \left[1 - e^{-a(r - r_e)}\right]^2 \qquad (5\text{-}13)$$

The Morse potential leads to an exact solution of the wave equation $H\psi_\lambda = \epsilon_\lambda \psi_\lambda$ in terms of Laguerre polynomials, but the Franck-Condon terms $\left[\int \psi_{\nu'} \psi_{\nu''} dr\right]^2$ converge so slowly that the calculation of the vibrational transition probabilities $R^2_{\nu',\nu''} = \bar{R}_e^2 \left[\int \psi_{\nu'} \psi_{\nu''} dr\right]^2$ becomes very cumbersome. In the above, $\bar{R}_e$ is the electronic transition moment.

Among the various suggestions, Bouigue[5-7], Fraser et al[5-8], Bates[5-9] and Wu[5-10], the suggestion of Wu concerning the possible application of the WKB (phase integral) method may provide a convenient solution. The WKB method has the advantage of being valid from any arbitrary potential function which is not as radically asymetric or has large gradients. It also automatically takes into account the shift of the internuclear distance (r_e). Furry[5-11] has indicated that in the zero'th vibrational level, the deviation of the WKB amplitudes are of the order of 2%. For higher vibrational levels, the agreement should gradually improve

The vibrational quantum numbers of the oscillator having the potential function V(x) are given by the phase integral

$$Q(x_1 x_2) = \int_{x_1}^{x_2} \frac{1}{\hbar} (2\mu)^{1/2} \left[|\epsilon_v - V(x)| \right]^{1/2} dx$$

$$= (v + 1/2)\, \pi \qquad (5\text{-}14)$$

where the integrand is the classical momentum of the oscillator, and x_1 and x_2 are the classical turning points (the momentum vanishes). Forms of V(x) have been given by Hulbert and Hirschfelder.[5-12]

The WKB method then gives the approximate solution to the vibrational wave function as exponential in the two regions outside of the turning points and a cosine function inside these limits. From the wave functions, the vibration transition probabilities as given approximately by

$$R^2_{v'v''} = \bar{R}_e^2 \left[\int \psi_{v'} \psi_{v''}\, dr \right]^2 \qquad (5\text{-}15)$$

can be obtained by computor methods.

One may conclude that it will be no simple task to get all the photo and (electron) impact transition probabilities required in the Boltzmann equation for the interaction of an rf source with a slightly ionized gas. Yet, this must be accomplished before this mathematical discipline, which can in principle describe the complicated state, is fruitful.

5.3 Excitation by Electron Impact*

The quantum mechanical solution of electron impact with an atomic specie has received considerable attention - at least for the case of the hydrogen atom. The hamiltonian for the two electrons involved is given by

*From Rev. Mod. Phys. 23, 199 (1956).

$$H = -\frac{\hbar^2}{2m}\left(\nabla_1^2 + \nabla_2^2\right) + \frac{e^2}{r_{12}} - \frac{e^2}{r_1} - \frac{e^2}{r^2} \qquad (5\text{-}16)$$

To obtain solutions to the wave equation ($H\Psi = \epsilon\Psi$), it is customary to utilize an approximation of the form

$$\Psi(\vec{r}_1, \vec{r}_2) = \left(\Sigma + \int\right) \psi_n(\vec{r}_1)\, F_n(\vec{r}_2) \qquad (5\text{-}17)$$

wherein the degeneracy between impact* and bound electron is conveniently ignored. The $\left(\Sigma + \int\right)$ takes into account both discrete and continuous states respectively. Substituting the approximate form of ψ in the wave equation, multiplying by the complex conjugate ψ^* and integrating over space, one obtains the infinite set of simultaneous equations

$$\left(\nabla_2^2 + k_n^2\right) F_n = \left(\Sigma + \int\right) U_{nm} F_m \qquad (5\text{-}18)$$

where

$$U_{nm} = \frac{2me^2}{\hbar^2} \int \psi_n^* \left(\frac{1}{r_{12}} - \frac{1}{r_2}\right) \psi_m \, d\vec{r}_1$$

and

$$k_n^2 = \frac{2m}{\hbar}(\epsilon - \epsilon_n)$$

Asymptotic solutions of the subsidiary functions (F_n) can then be obtained from the usual criteria that the wave function is well behaved, and that the interaction vanishes for large distance ($\vec{r}_2$); these have the form:

$$F_n \rightarrow \frac{e^{ik_n r_2}}{r_2} f_n(\theta_2, \phi_2) + e^{i\vec{k}_0 \cdot \vec{r}} \delta_{0n} \qquad (5\text{-}19)$$

*Subscript 2 denotes the impact electron.

For the solid angle increment $d\omega\ (\theta,\phi)$, the differential scattering cross section for excitation of state ψ_n is $I_n = |f_n\ (\theta,\phi)|^2$.

For fast impact electrons (i.e., when the energy of the impact electron is much greater than the binding energy of the bound electron) the perturbation is small, so that the Born approximation is valid. However, for the case of rf-breakdown, fast electrons are not involved, and the Born approximation is inappropriate. A weaker restriction than that of the small perturbation (the two-state approximation with close coupling) is to assume that except for those terms associated with the initial atomic state, the diagonal terms of the matrix U_{nn} predominate over the non-diagonal ones. Physically, this implies that the distortion of the initial and final "wave" of the impact electron by the mean static fields of the atom in both its initial and final state is included, but that intermediate states of the system are ignored. Under these conditions, the wave function and the set of differential equations become:

$$\left(\nabla^2 + k_0^2 - U_{00}\right) F_0 = 0$$

$$\left(\nabla^2 + k_n^2 - U_{nn}\right) F_n = U_{n0} F_0 \quad (n \neq 0)$$

$$\Psi = \psi_0\ (\vec{r}_1)\ F_0\ (\vec{r}_2) + \psi_n\ (\vec{r}_1)\ F_2\ (\vec{r}_2) \qquad (5\text{-}20)$$

Commencing from F_0, the equations can be solved by iteration, the distorted wave being given by the first term in the series.

The degeneracy between the impact and bound electrons can be taken into account in the two-state approximation by including terms which are

either symmetric or antisymmetric (denoted by superscript $\pm$) with respect to spatial interchange. The Ψ function and the wave equation then yield:

$$\Psi^{\pm} = \psi_0 (r_1) F_0^{\pm} (\vec{r}_2) \pm \psi_0 (r_2) F_0^{\pm} (\vec{r}_1) + \psi_n (\vec{r}_1) F_n^{\pm} (\vec{r}_2)$$

$$\pm \psi_n (\vec{r}_2) F_n^{\pm} (\vec{r}_1) \qquad (5\text{-}21)$$

$$\left(\nabla^2 + k_0^2 - U_{00}\right) F_0^{\pm} (\vec{r}_2) \pm K_{00} (r_2 r_2') F_0^{\pm} (r_2') d r_2'$$

$$= U_{n0} F_n^{\pm} \mp K_{0n} (\vec{r}_2, \vec{r}_2') F_n^{\pm} (\vec{r}_2') d \vec{r}_2' \qquad (5\text{-}22)$$

$$\left(\nabla^2 + k_n^2 - U_{nn}\right) F_n^{\pm} (\vec{r}_2) \pm K_{nn} (r_2, r_2') F_n^{\pm} (r_2') d r_2'$$

$$= U_{n0} F_0^{\pm} \mp K_{n0} (\vec{r}_2 \vec{r}_2') F_0^{\pm} (\vec{r}_2') d \vec{r}_2' \qquad (5\text{-}23)$$

where K_{00}, K_{0n} and K_{nn} are given by

$$K_{mn} = \psi_n^* (r_1) \psi_m^* (r_2) \left[k_n^2 - \frac{2m}{h^2} \left(\frac{e^2}{r_{12}} + \epsilon_0\right)\right] = K_{nm}^* \qquad (5\text{-}24)$$

In keeping with the two-state concept, K_{00} and K_{nn} represent additional distortions of the initial and final electron waves, while K_{0n} is a coupling contribution which induces transition between the two states. The differential cross section now takes the form

$$I_n (\theta, \phi) d\omega = \frac{1}{4} |f_n^- (\theta)|^2 + \frac{3}{4} |f_n^- (\theta)|^2 d\omega \qquad (5\text{-}25)$$

In this exchange and distorted wave approximation, where U_{0n} and K_{0n} are small, the iteration proceeds rapidly yielding, say for the process of elastic scattering

$$\left(\nabla^2 + k_0^2 - U_{00}\right) F_0^{\pm} \pm K_{00}(\vec{r}_2', \vec{r}_2) F_0^{\pm}(\vec{r}_2') \, d\, r_2' = 0 \quad (5\text{-}26)$$

and for inelastic scattering (for n^{th} state)

$$\left(\nabla^2 + k_n^2 - U_{nn}\right) F_n^{\pm} \pm K_{nn}(\vec{r}_2', \vec{r}_2) F_n^{\pm}(\vec{r}_2') \, d\, \vec{r}_2' = U_{n0} F_0^{\pm} \mp K_{n0}(\vec{r}_2', \vec{r}_2) F_1(\vec{r}_2') \, d\, \vec{r}_2' \quad (5\text{-}27)$$

Bates[5-13] and Seaton[5-14] have shown that in the case of collisions of slow electrons with atomic oxygen, additional terms must be inserted to allow for close coupling to intermediate states of the same configuration. If, further, the small energy difference between these states is ignored, the problem remains a tractable one.

The restriction appropriate to the several approximations can be listed as:

Distorted wave	$K_{00} = K_{nn} = K_{0n} = 0$; U_{0n} is small
Potential distorted exchange	$K_{00} = K_{nn} = 0$; K_{0n}, U_{0n} are small
Exchange and potential distorted exchange	K_{0n}, U_{0n} are small
Close Coupling without exchange	$K_{00} = K_{nn} = K_{0n} = 0$
Close coupling with exchange	None

From an examination of the results of the various disciplines for the relatively simple case of the hydrogen atom, it appears that the distorted two-state approximation is sufficiently accurate, provided that the exchange interaction is included. For transitions involving several states of nearly the same energy, coupling between states must be considered.

Insofar as the rf-breakdown is concerned, the interest is quite naturally in atomic and molecular species such as N_2, O_2, NO, O rather than atomic hydrogen. For these cases, it is more realistic to resort to measurement to obtain the necessary cross sections for excitation.

In "Atmospheric Processes", cross sections of only a few of these pertinent electron-neutral excitations (or de-excitations) were listed. These included:

From Seaton[5-15] (for atomic oxygen)

$^3P \rightarrow {}^1D$; $^1D \rightarrow {}^3P$

$^3P \rightarrow {}^1S$; $^1S \rightarrow {}^1D$

$^1S \rightarrow {}^3P$

for atomic nitrogen

$^4S \rightarrow {}^2D$; $^2D \rightarrow {}^2P$

$^4S \rightarrow {}^2P$ $^2D \rightarrow {}^4S$

From Perceval[5-16] (for atomic oxygen)

$^3P \rightarrow 3p\,{}^3P$

$^3P \rightarrow 3p\,{}^5P$

From Stewart[5-17]

0 - 0, 0 - 1, and 0 - 2 N_2^+ bands

$N_2\ (X'\ \Sigma_g^+) \rightarrow N_2^+\ (B^3\Sigma_u^+)$

This brief summation gives some idea of the dearth of data in the areas of electronic, rotational, and vibrational excitation of pertinent species (especially N_2, O_2 and the oxides) under electron impact.

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6. Radiation Losses in RF Excited Plasmas

The rate of energy loss by radiation is essential to the determination of breakdown threshholds. In particular, there are at least two types of radiative processes to be considered. The first and dominant process includes all microscopic interactions such as atomic and molecular radiative transitions and perhaps to some extent chemiluminescence. The theoretical determination of the transition probabilities was discussed in Section 5. Experimental determinations of radiative transitions in an excited gas whose ionization varies over a wide range are extremely difficult. Monitoring of the decay of microwave discharges is being done but data are meager. The basic difficulty is that at the low densities of the upper atmosphere where radiation can become more important than the collisional processes, the observations must be made over long optical paths. In the laboratory, the 3-body (2-body plus wall) effects mask the slower two-body radiative processes.

Upper atmosphere experiments on radiative loss processes are in general very expensive when the investigation provides the energy (rf) of excitation. However, several of these experiments have already been funded; that they are not equipped to give the maximum information on optical effects is unfortunate. The reason is simply that the dominent motivation of these experiments is enhanced electron density and its decay modes. Of course, natural air-glow and aurora can give some information, but here excitation is chiefly from electron impact, and the

distribution of excited states may vary widely from that of a stratum irradiated by an rf-beam because of the wide variations in the electron energy distributions.

The three high power density experiments currently in (near) operation are:

(1) The Bailey experiments in Australia which beams .5 megawatts of power at the gyro-frequency (about 1.5 mc/s) vertically incident at the ionosphere. The basic idea is simply to raise the electron temperature of a thin stratum absorption at the gyro-frequency being particularly effective. If three-body attachment ($e + O_2 + O_2$) is an important electron loss process, the electron density will increase (Bailey calculates a factor of 10) since the attachment cross section decreases with energy in this range. The principle "observation" will be the electron density as measured by a C-4 ionosonde.

(2) At Jicamarca, Peru, Ken Bowles is conducting high power density irradiation of the upper atmosphere at a frequency of 50 mc/s and a power of six megawatts. Evidently, NBS is aware of the possible importance of the enhanced "air-glow". However, and this is an important point, the tests are at present unrelated to the breakdown problem. For this reason, there will be no attempt to optimize the equipment for the purposes suggested herein. Furthermore, equipment for the several experiments will, if no action is taken, be completely different for each installation, and no steps will be taken to utilize the results of all the experiments in

furthering the fundamental understanding of the breakdown phenomenon.

(3) In Puerto Rico, a 1000 ft "dish" is being constructed for studies on incoherent back scatter from electrons (not fluctuations in refractive index) at a frequency of 400 mc/s. Again high power densities are available; in particular, the focusing of the beam is much better than that obtainable in the other experiments. However, the amount of absorption may be significantly decreased since the non-deviative absorption coefficient (k) is given by

$$k = \frac{2\pi N e^2 \nu}{mc\,\mu\,([\omega \pm |\omega_L|]^2 + \nu^2)} \qquad (6\text{-}1)$$

Only in the lower atmosphere are electron collision frequencies high enough ($\nu > \omega$) to make k basically independent of ω. Otherwise for $\omega > \nu$, k varies as $1/\omega^2$, and the absorption of the rf will be very small - - perhaps well beyond detection of radiative transitions. The Puerto Rico experiment is therefore not an optimum one for our purposes. Yet, if no action be taken here, no attempt will be made to employ photometric devices to monitor the experiments in the eventuality that optical information is available.

In addition to the microscopic character determined essentially by composition and state occupations, a plasma under an external EM field is described by the macroscopic field vectors. Under these imposed fields, inhomogeneities in the plasma can give rise to energy loss by radiation which may affect the electron density at and near breakdown. As evidenced

in the review, some theoretical work has been done on the problem, but this work has not been conclusive in establishing the importance of this mechanism to the breakdown phenomenon. Therefore, it is suggested that a theoretical study of the problem be made, commencing with a review and evaluation of past work. It seems likely that such action can place the mechanism in proper perspective.

7. Electron Energy Distribution Functions and the Boltzmann Equation

The time sequence of events leading to rf breakdown can only be fully described by the Boltzmann Equation. Unfortunately for real cases, a total solution of the problem which must include all the microscopic and macroscopic interactions is beyond our present computational capabilities and perhaps even our knowledge of the specific microscopic interactions involved. It is quite natural then that the total problem is greatly simplified by theoretical workers who make an effort to phrase the problem in such a manner that the problem is tractable. The question raised by these efforts (a subject often inadequately stated in the literature) is: what is the motivation and range of validity of the various approximations? In the remainder of Section 7, R. Papa has reviewed the various theoretical treatments, and delineation and/or evaluated their relation to the problem of rf breakdown. This material supplements Chapter XIII (Plasma Dynamics) in Atmospheric Processes.

7.1 The Mechanics of RF-Breakdown

If an electromagnetic field is impressed upon a partially ionized gas, the free charges will gain energy from the EM field by suffering collisions with neutral species and having their ordered oscillatory motion changed to random thermal motion. In the lower atmosphere, the free charges will exist initially not as electron-ion pairs; but as small ions, Aitken nuclei, and molecular ions (no thermal electrons exist very long since their lifetime at STP is 10^{-8} sec due to three body attachment

rate of 5×10^{-30} cm^6 sec^{-1}). These free molecular ions will eventually give rise to electrons through inelastic collisions with neutral species (collisions with neutrals in metastable states may be an effective electron production mechanism). In the upper atmosphere, the daytime ambient electron density varies from 10^3 particles per cm^3 in the D region to 10^5 particles per cm^3 in the F region.

As the electron temperature increases due to randomization of the electron oscillatory motion through elastic collisions with neutrals, there will be various electron energy loss processes such as:

(1) Collisions which can excite electronic levels of atoms and/or molecules

(2) Collisions which excite molecular vibrational levels (sub-excitation electrons)

(3) Collisions which excite molecular rotational levels (subexcitation electrons)

(4) Ionization of atoms and dissociative or non-dissociative ionization of molecules.

Processes in item (4) actually increase the number of electrons present and a knowledge of the energy distribution function of the secondaries produced by the primaries is necessary for a detailed calculation of the breakdown condition. The electron concentration may also increase through collisional and associative detachment. In the upper atmosphere, photodetachment is an additional electron production mechanism. Electron loss processes include dissociative, radiative and three-body recombination, and radiative and three-body attachment. A microscopic description of the interaction of an EM field with a gas (in terms of the electron energy distribution function) involves a knowledge of the cross sections for the following processes: elastic electron-neutral collisions, ionization and excitation collisions, radiative and collisional deactivation of excited states, radiative, three-body and dissociative attachment, electron-ion, ion-ion and three-body recombination, charge transfer, ion-atom exchange, and collisional and associative detachment. Even if a thorough knowledge of the various rates and cross sections as a function of energy permitted an elimination of some of the relatively unimportant processes, the formulation of the problem in terms of a Boltzmann equation for the electron energy distribution function is extremely complicated and includes the following effects:

(1) In the ambient atmosphere there are various macroscopic processes such as turbulent diffusion, winds, shears, and fluid transport forces on a region where the EM radiation is being absorbed.

(2) The neutral composition may change (by such processes as $N_2 + e \rightarrow N_2^+ + 2e$ and then $e + N_2^+ \rightarrow N + N$) even before breakdown is reached.

Most of the experimental work on RF-breakdown has consisted of studies with simple geometrical arrangements of EM fields in a microwave cavity, in which the breakdown condition may be expressed as a straightforward boundary value problem.

The gases employed in the laboratory experiments are usually either hydrogen or pure air, and a radioactive source is used as the external ionizing agency in place of the solar flux. In most of the laboratory experiments, electron loss by recombination may be neglected compared with loss by attachment and diffusion. Brown and MacDonald[7-1] have formulated the CW RF-breakdown condition for a particular gas for a specific EM mode in a microwave cavity in terms of (1) the characteristic value of the time independent continuity equation for the electron density

$$\nabla^2 N_0 + (\nu/D)\, N_0 = 0$$

where $\nu = \nu_i - \nu_a$

ν_i = Frequency of ionization due to electron impact.

ν_a = Frequency of attachment for electrons to neutrals.

D = Electron diffusion coefficient.

and (2) the high frequency ionization coefficient

$$\zeta = \nu/DE^2$$

where E = electric field amplitude.

The CW breakdown condition (neglecting recombination compared with attachment and diffusion) for the case of infinite parallel plates with a uniform electric field may be expressed as

$$\nu/D = 1/\Lambda^2$$

or

$$\zeta = 1/\Lambda^2 E^2$$

where $\Lambda = L/\pi$ = diffusion length

L = plate separation.

This breakdown condition is valid within the following three limits:

(1) The mean free path is less than any dimension of the cavity,

(2) The frequency is sufficiently high so that the electrons do not lose appreciable energy between cycles, and

(3) The average motion of the electrons resulting from the action of the field and of the collisions is sufficiently small so that the field does not sweep the electrons out of part of the cavity in each half cycle.

One of the most general conditions for RF breakdown may be described by the condition $dn_e/dt \geq 0$ where n_e is the electron density (n_e satisfied the continuity equation). Hartman[7-2] has investigated the validity of such an approach. In his study, a theoretical expression for the electron density was obtained as a function of the RF field amplitude. However, Hartman made the following simplifying assumptions:

(1) Ionization occurs as a result of single impacts between gas atoms and sufficiently fast electrons.

(2) The gas has an infinite volume, and

(3) Negative ions are not formed, so that the only mechanism for electron removal is recombination with positive ions.

His calculations show that, at a certain critical field strength (the breakdown field), the density of electrons rises sharply. His simple model leads to a breakdown phenomenon at field strengths not far from the observed values. The gases treated are helium and neon at frequencies of 3000 megacycles. In the ambient atmosphere, the condition $dn_e/dt \geq 0$ will also describe the CW RF breakdown criterion. However,

for pulsed RF, the breakdown criterion may not be expressed as the point at which the rate of gain of electrons (impact ionization and photo-ionization) equals the rate of loss of electrons. For a pulsed RF beam, once breakdown has occurred the electron concentration will increase at the rate $e^{\nu t}$, where $\nu = \nu_i - \nu_a - D/\Lambda^2$ and ν_i = frequency of ionization by electron impact

ν_a = frequency of attachment of electrons to neutrals

D = electron diffusion coefficient

Λ = diffusion length

and photoionization and recombination have been neglected. This buildup will occur for any pulse whose amplitude is equal to or greater than that of the CW breakdown field. The RF pulse can penetrate the region until the electron density has reached the plasma resonant density

$$n_p = m_e \omega^2/4\pi e^2$$

where ω = frequency of EM wave.

It may be concluded from the previous discussion that a knowledge of the time-dependent electron energy distribution function is sufficient to determine the breakdown condition for CW and pulsed RF power. The CW breakdown condition may be found by determining the electric field amplitude for which the electron density

$$n_e(t) = \int f_e(v,t)\, d^3 v$$

shows a sharp increase in magnitude. In the case of RF pulses, a large part of the RF power may be transmitted through the hot spot in the ambient atmosphere even though breakdown has occurred. Here the RF pulse will be reflected when the electron density reaches the plasma resonant density:

$$n_e = \int f_e(v,t)\, d^3 v \gtrsim n_p = m_e\omega^2/4\pi e^2$$

The determination of the electron energy (or velocity) distribution function of course involves a solution of the Boltzmann equation.

7.2 The Boltzman Equation

In the case of a partially ionized plasma, an adequate description of the energy distribution function of each species involves the solution of a series of coupled Boltzmann equations:

$$\frac{\partial f_e}{\partial t} + \vec{v}\cdot\nabla_r f_e - \frac{e}{m_e}\left[\vec{E} + \vec{v} \times \vec{B}\right] \cdot \vec{\nabla}_v f_e = B_{ee} + \sum_j B_{e\ ion_j} + \sum_k B_{en_k}$$

$$\frac{\partial (f_{ion})_\ell}{\partial t} + \vec{v}\cdot\vec{\nabla}_r (f_{ion})_\ell + \frac{Ze}{(m_{ion})_\ell}\left[\vec{E} + \vec{v} \times \vec{B}\right] \cdot \vec{\nabla}_v (f_{ion})_\ell$$

$$= B_{e\ ion_\ell} + \sum_{k=1}^{n-1} B_{ion_\ell\ ion_k} + \sum_j B_{ion_\ell\ n_j}$$

$$\frac{\partial f_{n_j}}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f_{n_j} = \sum_k B_{n_j\ ion_k} + B_{en_j} + \sum_{\ell=1}^{N-1} B_{n_j\ n_\ell}$$

where $(f_{ion})_\ell$ is the distribution function for the ℓ^{th} ion constituent, f_{n_j} is the distribution function for the j^{th} neutral constituent, the subscript e refers to electrons, subscript n to neutrals, and B_{mn} is the contribution to the collision integral for the distribution function of the m^{th} type of particles in collision with an n^{th} type. The electron, ion, and neutral equations are coupled through the collision integral terms, B_{mn}. The macroscopic fields which appear in these equations are self consistent, in that the electrons are not conceived of as interacting directly with one another. Rather, they produce an electromagnetic field (the field vectors $\vec{E}$ and $\vec{B}$ must satisfy Maxwell's equations) which is expressed as a function of position and time. The electromagnetic field then acts on each electron through its position and time coordinates. In order that such an approach

be valid, it is necessary that the macroscopic fields $\vec{E}$ and $\vec{B}$ remain constant during a collision (ion-electron, electron-neutral, ion-neutral). It is further assumed that three-body forces are negligible compared with two-body forces.

Most of the papers on the kinetic theory of microwave discharges in gases (Holstein[7-3], Margenau[7-4], Allis and Brown[7-5], Reder and Brown[7-6]) neglect ion-electron and electron-electron collisions. However, Ginzburg and Gurevich[7-7], whose work is butlined in sections 7.4 and 7.5 have discussed the conditions under which it is permissible to neglect electron-electron and electron-ion conditions in a partially ionized plasma. Processes such as photo-ionization and photo-detachment have never been considered in the literature as terms in the Boltzmann equation. Electron elastic collisions with neutrals are the most easily handled process. Expressions for the collision integral for electron-neutral collisions may be obtained in terms of the neutral density, temperature, elastic collision cross section, and zero order electron energy distribution function. The derivation of these expressions usually involves the assumptions that the electron suffers only a small change in speed on collision with a neutral (basically the factor is 1-m/m). More sophisticated treatments (such as those of Allis[7-8], which is presented in Section 7.4) take into account the recoil of the heavy molecule under elastic collisions with electrons. Almost all treatments on the interaction of microwave energy with a gas consider only the changes in the electron distribution function, where the ion and neutral distribution functions are assumed to remain Maxwellian.

Inelastic collisions between electrons and neutrals are accompanied by the excitation of rotational, vibrational or electronic levels and also by ionization. In addition, second-order impacts are possible, in which the energy of the excited state of the molecule is transferred to the incoming electron. However, an exact calculation of all these inelastic processes would be very complicated. The appropriate cross sections are known only in a few cases.

7.3 Spherical Harmonic Expansions*

The presence of the collision integral terms B_{mn} in the Boltzmann equation (which makes the value of f at $\vec{v}$ depend on the number of electrons at very different velocities) characterizes the equation as a non-linear partial differentio-integral equation, for which no general method of solution is known. It is customary to employ a perturbation technique, by expanding the function f, to obtain solutions to this equation. Enskog's[7-9] method consists in expanding the distribution function in powers of α, where the system is considered to be close to thermal equilibrium and departures from equilibrium are caused by some agent α:

$$f = f_0 + \alpha f^1 + \alpha^2 f^2 + \ldots\ldots$$

Unfortunately, this technique does not converge well for charged particles in an electric field.

For the case of electrons (but not ions) in an electric field, a spherical harmonic expansion of the distribution function does converge more rapidly:

$$f(\vec{r},\vec{v}) = \sum_{\ell} f^{\ell}(\vec{r},\vec{v})\, P_{\ell}\,(\cos\theta)$$

$$= f^0 + f^1 \cos\theta$$

where the electric field is taken to lie along the z-axis.

Two scalar quantities that may be obtained from the zero order distribution function are:-

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*Parts of Section 3. are taken from Allis[7-8].

the electron concentration:

$$n = \int_0^\infty f^0 \, 4\pi \, v^2 \, dv$$

the energy density:

$$n\bar{u} = \int_0^\infty u \, f^0 \, 4\pi \, v^2 \, dv$$

Two vector integrals that may be obtained from the first order distribution function are:

the particle flow:

$$\vec{\Gamma} = n\vec{v}_d = \int \vec{f}_1 \frac{4\pi}{3} v^3 \, dv$$

Total radial flow in velocity space:

$$G = \frac{4\pi}{3} v^2 \, \vec{a} \cdot \vec{f}_1$$

where $\vec{a} = e\vec{E}/m$ and the quantity G (the gain) represents the rate of flow of electrons across any energy value in the direction of increasing energy.

7.3.1 The Gradient Term $\vec{\nabla} \cdot (\vec{\nabla} f)$ in the Boltzmann Equation

Taking the polar axis along the direction of grad n and assuming that there is rotational symmetry about that axis so that the associated Legendre polynomials are not required,

$$\text{div}(\vec{v}\, f^{\ell}\, P_{\ell}) = \vec{v} \cdot (\text{grad}\, f^{\ell})\, P_{\ell} = v \frac{\partial f^{\ell}}{\partial z} \cos\theta\, P_{\ell}$$

$$= v \frac{\partial f^{\ell}}{\partial z} \frac{\ell P_{\ell-1} + (\ell+1) P_{\ell+1}}{2\ell + 1}$$

so that

$$\left.\begin{aligned} \text{div}(\vec{v}\, f) &= v \sum_{0}^{\infty} \frac{\partial}{\partial z}\left[\frac{\ell}{2\ell+1} f^{\ell-1} + \frac{\ell+1}{2\ell+3} f^{\ell+1}\right] \\ &= \frac{v}{3}\frac{\partial f^{1}}{\partial z} + v\cos\theta \frac{\partial}{\partial z}\left[f^{0} + \frac{2}{5} f^{2}\right] + \ldots \\ &= \frac{v}{3}\,\text{div}\,\vec{f}^{1} + \vec{v} \cdot \text{grad}\left[f^{0} + \frac{2}{5} f^{2}\right] + \ldots \end{aligned}\right]$$

In the last line the scalar function f^1 has been replaced by the vector $\vec{f}^1$, and this notation removes the initial restriction on the direction of the polar axis of P_1.

7.3.2 Electric Field Terms

In this section we take the polar axis of the spherical harmonics along the electric field $\vec{E}$. Using the expression for partial derivatives,

$$\left(\frac{\partial}{\partial v_z}\right)_{v_x\, v_y} = \cos\vartheta \left(\frac{\partial}{\partial v}\right)_{\vartheta,\phi} + \frac{\sin^2\vartheta}{v}\left(\frac{\partial}{\partial \cos\vartheta}\right)_{v,\phi}$$

we find

$$\text{div}_v\, \vec{a}\, f^{\ell}\, P_{\ell} = \vec{a} \cdot \text{grad}_v\, f^{\ell} P_{\ell} = a\cos\vartheta \frac{\partial f^{\ell}}{\partial v} P_{\ell} + \frac{a f^{\ell}}{v} \sin^2\vartheta \frac{\partial P_{\ell}}{\partial(\cos\vartheta)}$$

$$\text{div}_v \vec{a} f = a \sum \left[\frac{\ell}{2\ell - 1} v^{\ell-1} \frac{d}{dv} \frac{f^{\ell-1}}{v^{\ell-1}} + \frac{\ell + 1}{2\ell + 3} \frac{1}{v^{\ell+2}} \frac{d}{dv} v^{\ell+2} f^{\ell+1} \right] P_\ell$$

$$= a \left[\frac{1}{3v^2} \frac{d}{dv} v^2 f^1 + \left(\frac{df^0}{dv} + \frac{2}{5} \frac{1}{v^3} \frac{d}{dv} v^3 f^2 \right) \cos\theta + \ldots \right.$$

$$\left. = \frac{1}{3v^2} \frac{d}{dv} v^2 \vec{a} \cdot \vec{f}^1 + \vec{a} \cdot \vec{v} \left(\frac{1}{v} \frac{df^0}{dv} + \frac{2}{5v^4} \frac{d}{dv} v^3 f^2 \right) + \ldots \right]$$

in which we have again introduced the vector notation to eliminate the requirement on the polar axis P_1.

7.3.3 Magnetic Terms

Since

$$\text{div}_v(\vec{\omega}_b \times \vec{v}) = 0$$

we have

$$\text{div}_v(\vec{\omega}_b \times \vec{v}f) = (\vec{\omega}_b \times \vec{v}) \cdot \text{grad}_v f = \vec{v} \cdot (\vec{\omega}_b \times \text{grad}_v f)$$

The expansion involves the associated Legendre functions and the general term will not be given. The first two terms yield

$$\text{div}_v(\vec{\omega}_b \times \vec{v}f) = -\vec{v} \cdot (\vec{\omega}_b \times \vec{f}^1)/v$$

7.3.4 Expansion in Fourier Series

We shall assume an alternating electric field, $E \sim e^{j\omega t}$, since we can always obtain the direct-current formulae by setting $\omega = 0$ and replacing root-mean-square values by the dc values. The distribution function will then, in general, be a Fourier series in ωt:

$$f = \sum_{\ell} \sum_{m} f_m^{\ell} P_{\ell} e^{mj\,\omega t}$$

and

$$\frac{\partial f}{\partial t} = \sum_{\ell} \sum_{m} mj\,\omega\, f_m^{\ell} P_{\ell} e^{mj\,\omega t}$$

The exponential notation is very convenient for all linear equations. Unfortunately, we shall have to take products such as $\vec{a} f^{\ell}$ and the meaning of ~~such a~~ product is the product of the real parts and not the real part of the product. Performing this operation and then reintroducing the exponential notation, we have

$$2a\, f^{\ell} = a\left[f_{1r}^{\ell} + \left(2f_0^{\ell} + f_2^{\ell}\right) e^{j\,\omega t} + \sum_{m=2}^{\infty} \left(f_{m-1}^{\ell} + f_{m+1}^{\ell}\right) e^{mj\,\omega t}\right]$$

where f_{1r}^{ℓ} indicates the real part of f_1^{ℓ}.

7.3.5 The Component Equations

If the Boltzmann integral term B is decomposed as well as the electric and magnetic terms, one obtains an infinite set of equations of which the zero- and first-order ones are given below:

$$B_0^0 = \frac{v}{3} \operatorname{div} \vec{f}_0^1 + \frac{1}{6v^2} \frac{d}{dv}\left(v^2 \vec{a} \cdot \vec{f}_{1r}^1\right)$$

$$B_1^0 = j\omega\, f_1^0 + \frac{v}{3} \operatorname{div} \vec{f}_1^1 + \frac{1}{6v^2} \frac{d}{dv}\left[v^2 \vec{a} \cdot \left(2\,\vec{f}_0^1 + \vec{f}_2^1\right)\right]$$

$$\vec{B}_0^1 = v \text{ grad } \left(f_0^0 + \frac{2}{5} f_0^2\right) + \vec{a}\left[\frac{1}{2}\frac{df_{1r}^0}{dv} + \frac{1}{5v^3}\frac{d}{dv}\left(v^3 f_{1r}^2\right)\right] - \vec{\omega}_b \times \vec{f}_0^1$$

$$\left.\vec{B}_1^1 = j\omega \vec{f}_1^1 + v \text{ grad } \left(f_1^0 + \frac{2}{5} f_1^2\right) + \vec{a}\left[\frac{1}{2}\frac{d}{dv}\left(2f_0^0 + f_2^0\right) + \frac{1}{5v^3}\frac{d}{dv} v^3\left(2f_0^2 + f_2^2\right)\right] - \vec{\omega}_b \times \vec{f}_1^1\right\}$$

In the case of electrons (but not ions) the spherical harmonic expansion converges well and the terms in f^2 are negligible. The Fourier series also converges well at high frequencies ($\omega/\nu_c > \lambda/2$). The transition to low frequencies in which the second-harmonic terms become large has been studied by Margenau[7-4]; this case will not be considered here. We, therefore, simplify the above set of equations to

$$B_0^0 = \frac{v}{3} \text{ div } \vec{f}_0^1 + \frac{1}{6v^2}\frac{d}{dv}\left(v^2 \vec{a} \cdot \vec{f}_{1r}^1\right)$$

$$B_1^0 = j\omega f_1^0 + \frac{v}{3} \text{ div } \vec{f}_1^1 + \frac{1}{3v^2}\frac{d}{dv}\left(v^2 \vec{a} \cdot \vec{f}_0^1\right)$$

$$\vec{B}_0^1 = v \text{ grad } f_0^0 + \frac{\vec{a}}{2}\frac{df_{1r}^0}{dv} - \vec{\omega}_b \times \vec{f}_0^1$$

$$\vec{B}_1^1 = j\omega \vec{f}_1^1 + v \text{ grad } f_1^0 + \vec{a}\frac{df_0^0}{dv} - \vec{\omega}_b \times \vec{f}_1^1$$

The divergence of the alternating flow $\vec{\Gamma}_1$ produces an alternating electron density n_1. The alternating flow $\vec{\Gamma}_1$ is driven by $\vec{a}\, n_0$ and, hence, n_1 exists in virtue of a divergence in $\vec{a}\, n_0$. Since $\vec{a}$ is divergenceless, n_1 is produced by the interaction of an alternating electric field and a concentration gradient in the same direction. However, if such situations do not occur, it may be assumed that n_1 and f_1^0 are

neglibible. The four component equations may then be reduced to three simpler equations:

$$B_0^0 = \frac{\vec{v}}{3} \operatorname{div} \vec{f}_0^1 + \frac{1}{6v^2} \frac{d}{dv}\left(v^2 \vec{a} \cdot \vec{f}_{1r}^1\right)$$

$$\vec{B}_0^1 = \vec{v} \cdot \operatorname{grad} f_0^0 - \vec{\omega}_b \times \vec{f}_0^1$$

$$\vec{B}_1^1 = j\omega \vec{f}_1^1 + \vec{a} \frac{df_0^0}{dv} - \vec{\omega}_b \times \vec{f}_1^1$$

7.4. The Collision Integrals*

7.4.1 Elastic Collisions Between Electrons and Neutral Particles

7.4.1.1 Geometry of a Collision

Consider a collision between an electron, denoted by lower case letter, and a neutral molecule, denoted by capital letters. In a collision, the positions of these particles do not change appreciably but their velocities change; we shall denote by primes the velocities of particles about to be scattered into $d^3 v\, d^3 V$ about the velocities $\vec{v}$ and $\vec{V}$. We do not consider the changing velocities during the collision but only the velocities before and after the particles are within range of their interaction forces. During the collision the velocity of the center of gravity, $\vec{v}_g$, remains fixed and if the collision were elastic, the relative velocity c has the same magnitude before and after but has changed in direction by an angle χ_g, called the scattering angle in the center of gravity system. The center of gravity divides the relative velocity inversely as the masses, so that the velocities of electron and molecule remain on two spheres in velocity space centered on the velocity of the center of gravity and of radii Mc/(M+m) and mc/(M+m). The radius of the molecule's sphere is very small compared with that of the electron, so that it can almost be considered as a point. If the molecule were at rest before the collision, the origin of velocity coordinates should be taken at $\vec{V}'$

$$(\vec{v} - \vec{V}')^2 = c^2 - \frac{2Mm}{(M+m)^2} c^2 (1 - \cos \chi_g)$$

or if $V' = 0$

$$v'^2 - v^2 = \frac{2Mm}{(M+m)^2} v'^2 (1 - \cos \chi_g)$$

*Parts of Section 5 are from Allis[7-8] and Ginzburg and Gurevich[7-7].

This last expression gives the energy loss by an electron in striking a molecule at rest. This energy goes into the recoil energy of the molecule. For electrons it is small and we can write it in the form

$$-\frac{\Delta v}{v} = \frac{Mm}{(M+m)^2}(1 - \cos \chi_g)$$

Another useful relation from the triangle $\vec{V}'$, $\vec{v}_g$, $\vec{v}$ gives the scattering angle χ_0 in the laboratory system in terms of the scattering angle χ_g in the center of gravity system. It is

$$\sin^2 \chi_0 = \frac{M^2 \sin^2 \chi_g}{M^2 + m^2 + 2M\, m \cos \chi_g}$$

The scattering function $\sigma(\chi)$ is deduced from the force law as a function of χ_g and c, and is measured experimentally in terms of χ_0 and v', and it is always necessary to reduce one to the other. The dependence of σ on the azimuthal angle ψ is never known and it will be assumed that σ does not depend on ψ. The combination $c\sigma$ always enters the equations and we shall use the sysbol $\rho(c) = c\sigma$ for it (ρ has the dimensions of a volume per second per steradian).

7.4.1.2 Reduction of the Collision Integral Term

The rate of change of the number of particles in a velocity volume element d^3v is given by taking the difference between the number of particles scattering into that volume element and the number of scattered out

$$B\, d^3 v = \int F(\vec{V}')\, f(\vec{v}')\, \rho\, d^2\Omega\, d^3V'\, d^3v' - \int F(\vec{V})\, f(\vec{v})\, \rho\, d^2\Omega\, d^3V\, d^3v$$

ρ is the same in both these integrals and $d^2\Omega$ is the solid angle of the scattering sphere. $F(\vec{V}')\ d^3V'$ and $f(\vec{v}')\ d^3v'$ are the numbers of particles in position to scatter into the volume elements d^3V and d^3v, and the integrals are to be taken over all scattering angles and all molecular velocities. By Liouville's theorem

$$d^3v'\ d^3V' = d^3v\ d^3V$$

so that the two integrals may be combined into one

$$B = \int [F(\vec{V}')\ f(\vec{v}') - F(\vec{V})\ f(\vec{v})]\ \rho\ d^2\Omega\ d^3V$$

which is the conventional way of writing Boltzmann's collision integral. This form is, however, rarely convenient, since $F(V')$ contains V' and not V and it is not easy to inegrate it with respect to V. It is more convenient to keep dV' and convert d^3v' into d^3v. We wish to determine the volume element d^3v' so that, for given $\vec{V}'$ and χ_g, the final velocity after the collision ends up in d^3v. As all angles are held constant and $\vec{V}'$ is held fixed, the volume element d^3v' must be exactly similar in shape to d^3v, but all linear dimensions increased in the ratio $|(\vec{v}' - \vec{V}')/(\vec{v} - \vec{V}')|$ so that

$$d^3v' = (|\vec{v}' - \vec{V}'| \Big/ |\vec{v} - \vec{V}'|)^3\ d^3v$$

This leads to the expression:

$$B = \int F(V')\ f(v')\ (|\vec{v}' - \vec{V}'| \Big/ |\vec{v} - \vec{V}'|)^3\ \rho\ d^2\Omega\ d^3V' - \int F(V)\ f(v)\ \rho\ d^2\Omega\ d^3V$$

7.4.1.3 Expansion in Spherical Harmonics

We now substitute for $f(\vec{v})$ its expansion in spherical harmonics and for $f(\vec{v}')$ the corresponding series in terms of θ'. Using the addition theorem for spherical harmonics, we have

$$f(\vec{v}') = \sum f^{\ell} \left[P_{\ell}(\theta)\, P_{\ell}(\chi_0) + 2 \sum_{1}^{\ell} \frac{(\ell - m)!}{(\ell \quad m)!} P_{\ell}^{m}(\theta)\, P_{\ell}^{m}(\chi_0) \cos \vec{m}(\theta - \psi) \right]$$

As ρ has been assumed to be independent of the azimuthal angle ψ, the term in $\cos m(\theta-\psi)$ goes out on integrating, so that the Boltzmann integral for the coefficient of $P(\theta)$ becomes

$$B^{\ell} = \int \left(\frac{|\vec{v}' - \vec{V}'|}{|\vec{v} - \vec{V}'|} \right)^3 F(\vec{V}')\, f^{\ell}(v')\, P_{\ell}(\chi_0)\, \rho\, d^2\Omega\, d^3V$$

$$- \int F(V)\, f^{\ell}(v)\, \rho\, d^2\Omega\, d^3V$$

So far the equations have been perfectly general, but we must now make assumptions which in practice restrict their application to electrons, or at most to light ions in a heavy gas. We shall assume that the mass ratio m/M is zero, so that $v' = v$, and that $F(\vec{V})/n_g$ is a δ-function. Then,

$\ell = 0 \qquad B^0 = 0$

$\ell \neq 0 \qquad B^{\ell} = -f^{\ell}(v) n_g \int \rho(v,\chi_0)[1 - P_{\ell}(\chi_0)]\, d^2\Omega = -\nu_{c\ell} f^{\ell}$

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The "collision frequencies" ν_{cl} introduced above are defined in the laboratory reference system. Introducing an arbitrary definition ν_{c0}, the first three collision frequencies are

$$\nu_{c0} = \int n_g \rho \, d^2\Omega$$

$$\nu_{c1} = \int n_g \rho (1 - \cos \chi_0) \, d^2\Omega$$

$$\nu_{c2} = \int n_g \rho \frac{3}{2} \sin^2 \chi_0 \, d^2\Omega$$

The total collision frequency is ν_{c0}. The integral is generally improper as ρ has a singularity in the forward direction. The apertures of experimental apparatus prevent observations from covering this singularity, so that the quantities tabulated as experimental "probabilities of collision" are expressed in terms of an incomplete integral.

The collision frequency for "momentum transfer", ν_{c1}, is of great importance in transport theory and the simpler symbol ν_c is used for it. It is a weighted collision frequency in which the backward scattering counts double, right angle scattering has the weight one, and forward scattering is not counted. The frequency ν_{c2} might well be called the "scattering" frequency; here right-angle scattering has the weight 3/2 and neither forward nor backward scattering is counted.

7.4.1.4 Recoil of the Heavy Molecule

The conclusion that $B^0 = 0$ means that collisions do not change the energy distribution, and it follows from the assumption that the molecules were elastic

and infinitely heavy. In fact, it may take many thousand collisions to change the energy of an electron appreciably whereas exactly one "collision" defined by the frequency ν_{cl} will cancel its momentum. Nevertheless, many thousand collisions do occur and we must introduce the higher-order terms for $\ell = 0$.

We first relax the restriction to infinitely heavy molecules so as to allow the molecule to recoil under electronic impact ($v' > v$) but continue to neglect thermal motion ($V' = 0$). Then $c = v'$ and

$$v'^2 - v^2 = v'^2 \frac{2Mm}{(M+m)^2} (1 - \cos \chi_g) = \mu v'^2$$

Let us expand in terms of μ which is assumed to be small

$$v'^3 \rho(\vec{v}') f(\vec{v}') = v^3 \rho f(\vec{v}) + \mu v^2 \frac{d(v^3 \rho f)}{d(v^2)}$$

Integrating the expression for B^0 with respect to $d^3 V'$ for $T_g = 0$ gives

$$B^0 = \int \frac{v'^3}{v^3} n_g \rho(v') f^0(v') - n_g \rho f^0(v) \; d^2\Omega$$

$$= \int \frac{\mu}{v} n_g \frac{dv^3 \rho f^0}{d(v^2)} d^2\Omega = \frac{2Mm}{(M+m)^2} \frac{1}{v} \frac{d}{d(v^2)} (v^3 \nu_{gl} f^0)$$

where ν_{gl} is the same as ν_{cl} but in the center of mass instead of the laboratory system. The relation between these two depends on how the cross-section ρ depends on χ_g. It is given in first order by

$$\nu_c = \frac{M}{M+m} \nu_{gl}$$

Hence,

$$B^0 = \frac{2m}{M+m}\frac{1}{v}\frac{d}{d(v^2)}\left(v^3 \nu_c f^0\right)$$

Next we take account of the Maxwell distribution of the molecules $F(\vec{V}')$, which was previously replaced by a delta function, and this will be done in a somewhat qualitative way. In integrating over $d^3 V'$ the point V' moves over a Maxwell distribution centered about V' = 0. For fixed v and scattering angle X_g, this means that $\vec{v}'$ moves about $\vec{v}'_0$, its value for V' = 0, by amounts of the order $\vec{V}'$.

$$\vec{v}' = \vec{v}'_0 + \mathcal{O}(\vec{V}')$$

and

$$f(\vec{v}') = f(\vec{v}'_0) + \mathcal{O}(V'^2)\frac{\partial f}{\partial(v'^2)}$$

when this correction is introduced into the expression for the collision integral, one obtains

$$B^0 = \frac{2m}{M+m}\frac{1}{v}\frac{d}{d(v^2)}\left[v^3 \nu_c \left(f^0 + \mathcal{O}(V'^2)\frac{df^0}{d(v^2)}\right)\right]$$

But we know that if the electrons have a Maxwell distribution at the same temperature as that of the gas, collisions will not change this distribution and, therefore, this last equation must give zero if f is a Maxwell distribution. This determines the factor $\mathcal{O}(V'^2)$ to be equal to $2k T_g/m$, and we have

$$B^0 = \frac{2m}{M+m}\frac{1}{v}\frac{d}{d(v^2)}\left[v^3 \nu_c \left(f^0 + \frac{2k T_g}{m}\frac{df^0}{d(v^2)}\right)\right]$$

for the rate of change of the energy distribution of electrons of mass m owing to elastic collisions with molecules of finite mass M having a temperature T_g.

7.4.2 Elastic Collisions of Electrons with Ions

To describe elastic collisions between electrons and ions one can employ the foregoing general expressions for the integral of elastic collisions between an electron and neutral particles, without modification, since the only assumption made in their derivation was $m \ll M$. It is necessary only to calculate the number of collisions between the electrons and the ions, $\nu_i(v)$. For this purpose one substitutes in the collision integral the Rutherford formula for the differential effective cross section for the scattering of an electron by an ion. We then have

$$\nu_i(v) = 2\pi N_i v (e^2/2mv^2)^2 \int_{\theta_{min}}^{\pi} \frac{1 - \cos\theta}{\sin^4 \frac{\theta}{2}} \sin\theta \, d\theta$$

$$= 2\pi N_i \frac{e^4}{m^2 v^3} \ln\left(1 + \cot^2 \frac{\theta_{min}}{2}\right)$$

where N_i is the concentration of the ions, which for simplicity are assumed to be singly-charged.

If we consider the scattering of an electron by a free ion, then integration should be carried out from 0 to π (i.e., $\theta_{min} = 0$) and the collision frequency diverges logarithmically at small θ. In a plasma, however, the ions are not entirely free: as a result of interaction between the ions and the electrons, the field of each ion, under equilibrium conditions, has a Coulomb character only to distances on the order of the Debye radius D:

$$D = \left[\frac{kTkT_e}{4\pi e^2 N(kT + kT_e)} \right]^{1/2}$$

where T = ion temperature and

t_e = electron temperature.

At distances greater than D, the Coulomb field of the ion drops off as a result of screening. Consequently, D is the maximum distance at which a substantial interaction between the electron and the ions still takes place, i.e., the maximum impact parameter. It can be used to express the minimum scattering angle:

$$\theta_{min} = 2 \tan^{-1}(e^2/mv^2D) \approx 2e^2/mv^2D$$

Therefore:

$$\nu_i(v) = 2\pi N_i \frac{e^4}{m^2v^3} \ell n \left(1 + \frac{D^2m^2v^4}{e^4}\right)$$

It is important that $D^2(kT_e)^2 e^{-4} \sim k^3 T_e^3 e^{-6} N^{-1}$ always be a large quantity in the cases of interest to us. This means that the second term in the logarithm is always the principal term. Consequently, the principal contribution to the number of collisions between the electron and the ions is made by the weak scattering -- scattering by small angles. In one such collision, the change in either the energy or in the electron momentum is insignificant. In fact, the fraction of the energy lost by the electron when scattered by an angle ϑ is $\delta_k = 2m(1 - \cos\vartheta)/M$. Considering that the principal role is played by collisions that lead to the scattering by a small angle, on the order of ϑ_{min}, we find

$$\delta_k \sim \delta_{k_{min}} = \vartheta^2_{min} \frac{m}{M} = \frac{m}{M}\left(\frac{e^2 N^{1/3}}{kT_e}\right)^3 \ll 1$$

Analogously, the change in the momentum is

$$\delta_p = |p - p_1| \,/\, |p| \sim \delta_{p\ min} = \left(\frac{e^2 N^{1/3}}{kT_e}\right)^3 \ll 1$$

It must be emphasized that although the change in the momentum in one impact is small, the change in the energy is considerably smaller: $\delta_k/\delta_p \sim m/M$.

Scattering by large angles adds to the number of collisions only a term of order unity, which is small compared with the main logarithmic term. A similar correction in equilibrium plasma results from an exact solution of the problem of scattering in a Debye field.

Inelastic collisions between electrons and ions, which lead to their excitation and multiple ionization, do not differ at all from inelastic collisions with neutral particles, considered above. However, owing to the large values of the maximum elastic impact parameter (ϑ), the role of inelastic collisions is greatly reduced. Collisions of electrons accompanied by bremsstrahlung, which is of importance at high electron energies, will not be considered here.

7.4.3 Electron-Electron Collisions

The principal role in a collision between an electron and ions, as seen above, is played by long-range collisions, which lead to weak scattering. Both the energy and the momentum of the electrons are changed only slightly by one such collision. This is the consequence of the singularity of the Coulomb interaction and therefore pertains

not only to collisions between electrons and ions, but also to collisions between electrons and electrons. The difference lies only in the fact that the fraction of the energy and the fraction of the momentum lost by the electron when colliding with another electron is of the same order $\delta_k/\delta_p \sim 1$, whereas in the collision with the ion $\delta_k/\delta_p \sim m/M$.

Thus, in considering the integral of inter-electron collisions, one can use the differential expressions derived earlier for B. In addition, we can integrate this expression over the scattering angles $d\Omega$ (using the fact that $\sigma(\theta,u)$ has a sharp maximum at $\theta \sim 0$). We then find that the integral of the collisions between the electrons are satisfied by the expression

$$B_{ee} = - \operatorname{div} j_v$$

where

$$j_v = \frac{1}{2\pi} \int dv_1 \nu(u) \{u^2(f(v) \operatorname{grad}_{v_1} f(v_1) - f(v_1) \operatorname{grad}_v f(v))$$

$$- u[f(v) (u \operatorname{grad}_{v_1} f(v_1)) - f(v_1)(u \operatorname{grad}_v f(v))]\}$$

and v_1 is the velocity of the particle with which the electron collides and $u = |\vec{v} - \vec{v}_1|$, $\nu(u)$ is the number of collisions, where v must be replaced by u and N_i by $N_e = N$; account is also taken of the fact that the scattering particles are electrons, i.e., that $F(v_1) = f(v_1)$.

Let us consider now B_{ee} - the integral of inter-electron collisions for the function f_0, by putting $f = f_0(v)$ into the expression for the collision integral and integrating over the angles, one obtains:

$$B_{ec} = -\frac{1}{v^2}\frac{\partial}{\partial v}\{v^2[A_1(f_0)\, vf_0 + A_2(f_0)\frac{\partial f_0}{\partial v}]\}$$

where

$$A_1 = -\frac{1}{N}\int dv_1\, v_1 \frac{\partial f_0}{\partial v_1}(1 - \cos^2\theta_1)\,\nu(u)$$

$$= \frac{4\pi\nu(v)}{N}\int_0^v v_1^2 f_0(v_1)\, dv_1$$

$$A_2 = \frac{1}{N}\int v_1^2 f_0\,(1 - \cos^2\theta_1)\,\nu(u)\, dv_1$$

$$= \frac{4\pi\nu(v)}{3N}\int_0^v v_1^4 f_0(v_1)\, dv_1 + v^3\int_v^\infty v_1 f_0(v_1)\, dv_1$$

Here θ_1 is the angle between v and v_1, $u = |v - v_1|$; when integrating over the angles we neglected the variation of the logarithmic term in $\nu(u)$ (compared with the variation of the principal term $\sim 1/u^3$). For fast electrons, whose velocity v is much greater than the average velocity of the plasma electrons, the coefficients A_1 and A_2 assume a simple form: $A_1 = \nu(v)$, and $A_2 = 2\bar{K}\,\nu(v)/3m$, where $\bar{K}$ is the average energy of the scattered electrons (in the case of a Maxwellian electron velocity distribution, $2\bar{K}/3m = kT_e/m$).

7.4.4 Inelastic Electron-Neutral Collisions

Inelastic collisions between electrons and neutral particles are accompanied by the excitation of rotational, vibrational, or electronic levels, and also by ionization. In addition, so called second-order impacts are possible, in which the energy of the excited state of the molecule is transferred to the incoming electron. An exact cal-

culation of all these inelastic processes is quite complicated; in addition, their cross sections are known only in a few cases. Therefore there is no complete theory of inelastic collisions in which the problem is solved as accurately as in the case of elastic collisions. In spite of this fact, it is possible to analyze relatively simply two important limiting cases; specifically, we consider cases when the electron energy is considerably greater than the energy of the excited level, or the ionization energy ($K \gg \hbar\omega$), and when, to the contrary, the electron energy is only slightly higher than the excitation energy ($K - \hbar\omega \ll K$ and positive).

In the former case the expression for the integral of inelastic collisions is found in the same manner as for elastic collisions. It is merely necessary to consider that the energy lost by the electron in inelastic impact is consumed essentially in excitation of the molecule, and this is connected with a transfer of energy $\hbar\omega$ (thus, $v' - v = h\omega/mv$); in this case the neutral particle simply goes from the ground state into the excited state. We then have

$$(B^0)_{\text{inelastic}} = -\frac{1}{2v^2}\frac{\partial}{\partial v} \quad v^2 r_\omega \quad \frac{kT_\omega}{m}\frac{\partial f_0}{\partial v} + v f_0$$

$$(B^1)_{\text{inelastic}} = \nu_\omega f_1$$

Here ν_ω is the number of inelastic collisions accompanied by the excitation of a quantum $\hbar\omega$ (as we shall call the transfer to the molecule of an energy $\hbar\omega$, consumed in excitation of some level)

$$\nu_\omega(v) = v(N_m^0 + N_m^{ex}) \int \sigma_\omega(v, \vartheta)\ (1 - \cos\vartheta)\ d\Omega$$

where $\sigma_\omega(v,\vartheta)$ is the differential effective scattering cross section in inelastic collision, N_m^0 and N_m^{ex} are the number of molecules in the ground and excited states respectively. Furthermore, $r_\omega(v)$ is the fraction of the energy lost per unit time by the electron to excitation of a quantum $\hbar\omega$

$$r_\omega(v) = \frac{2\hbar\omega}{mv^2}\,(N_m^0 - N_m^{ex})\,v \int \sigma_\omega(v,\vartheta)\, d\Omega$$

and T_ω is the effective temperature

$$T_\omega = \frac{\hbar\omega}{2k}\,\frac{N_m^0 + N_m^{ex}}{N_m^0 + N_m^{ex}}$$

It is important to emphasize that in the case when the quantum $\hbar\omega$ is small not only compared with the electron energy, but also compared with the energy of neutral particles ($\hbar\omega \ll kT$), and if the neutral particles have a Boltzmann distribution $N_m^{ex}/N_m^0 = \exp(-\hbar\omega/kT)$ (i.e., if the collisions with the electrons do not substantially change the number of excited molecules), then the effective temperature T_ω is equal to the molecule temperature T_1.

In the second limiting case, when the electron energy is slightly greater than the excitation energy, the colliding electron merely goes from the region of large energies into the region of small energies ($K \sim 0$). Therefore at large energies

$$(B^0)_{inel} = \nu_\omega f_0 = N_m v \int \sigma_\omega(v,\vartheta)\, d\Omega$$

$$(B^1)_{inel} = -\nu_\omega f_1$$

where ν_ω is the total frequency of the excitations of the level $\hbar\omega$ (it is assumed that $N_m^{ex} \ll N_m^0$). The fact that the electron cannot simply vanish but goes into the region of small energies ($K \sim 0$) is taken into account here by adding to the equation for f_0 a δ-function source of elections, $-Q\delta(0)/4\pi v^2$,

where $Q = dN/dt = 4\pi \int_{\hbar\omega}^{\infty} \nu_\omega f_0 v^2 dv$.

We note in conclusion that in the general case the characteristic dependence of the total cross section of the inelastic collision on the electron energy is such that the total cross section $\sigma(v)$ vanishes when $K < \hbar\omega$, then it increases, reaching a maximum $K \sim (3 \text{ to } 5)\hbar\omega$, and then starts to diminish slowly. Since in the average impact $K \gg \hbar\omega$ and the electron loses an energy $\hbar\omega$, the electron in the most probable inelastic collisions loses a small fraction of its energy.

We note also that in those cases when not one but several levels $\hbar\omega$ can be excited, we have $B = \sum_i B_{\omega i}$. It must be taken into account also that some of the inelastic collisions -- ionization and the effective recombination (recombination, capture of electron by a molecule, etc) -- are accompanied by a change in the number of electrons in the plasma. It is therefore necessary to add in the collision integral for the function f_0 the terms

$$-\nu_r(v)\, f_0 + \int_{\sqrt{\frac{2\hbar\omega_i}{m}}}^{\infty} \nu_{ion}(v', v)\, f_0(v')\, v'^2\, dv'$$

the first of which describes the effective recombination, and the second describes ionization. Here $\nu_r(v)$ is the total recombination frequency

and $\nu_{ion}(v',v) = N_m v' \int \sigma_{ion}(v',v,\vartheta)\, d\Omega$

is the ionization frequency, i.e., the number of ionizations produced per second by electrons of velocity v', which lead to the appearance of a new electron of velocity v, where $\hbar\omega_i$ is the ionization energy. These terms usually do not exert a noticeable influence on the form of the distribution function; they do determine, however, the concentration of the electrons in the plasma.

7.5. Solutions of the Boltzmann Equation

If a spherical harmonic expansion of the electron distribution function is made $\left(\text{i.e., } f = f_o + \frac{\vec{V}.\vec{f}_1}{V}\right)$, the two Boltzmann equations for the zero and first order distribution functions (taking into account electron-electron, elastic electron-ion and elastic and inelastic electron-neutral collisions) are:

$$\frac{\partial f_o}{\partial t} + \frac{v}{3}\,\mathrm{div}_r\,\vec{f}_1 + \frac{e}{3mv^2}\frac{\partial}{\partial v}\left(v^2\vec{E}.\vec{f}_1\right) = \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta(\nu^{ELAS}+\nu_i)\left[\frac{kT}{m}\frac{\partial f_o}{\partial v} + vf_o\right]\right\}$$

$$+ B_{en}^{INELAS}(f_o) + B_{ee}(f_o)$$

$$\frac{\partial \vec{f}_1}{\partial t} + v\,\mathrm{grad}_{rfo} + \frac{e\vec{E}}{m}\frac{\partial f_o}{\partial v} + \frac{e}{mc}\left[\vec{H}_o \times \vec{f}_1\right] = -\nu(v) f_1 - B_{ee}(f_1)$$

Here,

$$B_{ee}(f_o) = +\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[A_1(f_o)vf_o + A_2(f_o)\frac{\partial f_o}{\partial v}\right]\right\}$$

$$A_1 = \frac{4\pi\nu(v)}{N}\int_o^v v_1^2 f_o(v_1)\,dv_1$$

$$A_2 = \frac{4\pi\nu(v)}{3N}\left\{\int_o^v v_1^2 f_o(v_1)\,dv_1 + v^3\int_v^\infty v_1 f_o(v_1)\,dv_1\right\}$$

$$\delta = \frac{2m}{M}$$

ν^{EL} = electron-neutral elastic collision frequency

ν_i = electron-ion elastic collision frequency

$\nu = \nu^{ELAS} + \nu_i + \nu^{INELAS}$

ν is the total number of electron collisions, where $\nu^{INEL}(v) =$

$$\Sigma\, v\left(N_{MOL}^{IONIZ} + N_{MOL}^{EXCITED}\right)\int \sigma_{v_i}(v,\theta)(1-\cos\theta)\,d\Omega$$

$B_{en}^{INELAS}(f_o)$ is the collision integral for the function f_o, which describes inelastic collisions between electrons and molecules.

It should be brought out that the electron energy relaxation time is always much greater than the momentum relaxation time. This implies that the relaxation time is much greater for the function f_o than for the function f_1. As a consequence, the function f_o always changes more slowly with time than the function f_1. Hence, when the equation for f_1 is integrated with respect to time, the function f_o can be considered time independent to a first approximation.

In the equation for the function f_o, the last term on the right side of the equation ($B_{ee}(f_o)$), which represents electron-electron collisions, is of the order $\nu_e f_o$, where ν_e is the inter-electron collision frequency. The remaining terms, which describe the collisions between electrons and heavy particles, is of the order $\delta\nu f_o$, where $\delta = {}^{2m}/_{M}$, $\nu = \nu^{EL} + \nu_i + \nu^{INELAS}$. Hence, the relation between ν_e and $\delta\nu$ determine the form of the function f_o. A highly ionized plasma is characterized by the condition $\nu_e \gg \delta\nu$; and a weakly ionized plasma by the condition $\nu_e \ll \delta\nu$ (in a fully ionized plasma, $\nu_e \gg \delta\nu_i$).

7.5.1 Highly Ionized Plasma

In a highly ionized plasma ($\nu_e \gg \delta\nu$), the form of the zero order distribution function f_o is set by the inter-electron collisions. The solution of the general Boltzmann equations may be obtained by the method of successive approximations: $f_o = f_{oo} + f_{o1} + \ldots$ where the zero order approximation includes only inter-electron collisions. If the Boltzmann equations are solved for the zero order approximation, f_{oo}, one obtains a Maxwellian distribution function in the case of a homogeneous

plasma:

$$f_{oo} = N\left(\frac{m}{2\pi kT_e}\right)^{3/2} \exp\left\{-\frac{mv^2}{2kT_e}\right\}.$$

The Maxwellian distribution becomes established within a time $1/\nu_e$. When $\nu_e \gg \delta\nu$ this process is much faster than the process of transfer of energy to heavy particles, this implies that f_o is close to being Maxwellian.

If the expression $f_o = f_{oo} + f_{01} + \ldots$ is substituted into the Boltzmann equation for f_o, and then both sides are multiplied by $4\pi v^2$ and integrated with respect to v, one obtains:

$$\frac{d}{dt}\left(4\pi \int_o^\infty v^2 f_{oo}\, dv\right) + \int_o^\infty 4\pi v^2 B_{en}^{INELAS}(f_{oo})\, dv = 0$$

If the Maxwellian distribution is taken for f_{oo}, then

$$\frac{dN_e}{dt} + [\nu_{rec} - \nu_{ion}] N_e = 0$$

where

$$\nu_{ion} = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT_e}\right)^{3/2} x \int_o^\infty \int^\infty \sqrt{\frac{2h\nu_i}{m}}\; v^2(v')^2 \nu_{im}(v,v') \exp\left\{-\frac{mv'^2}{2kT_e}\right\} dv\, dv'$$

and

$$\nu_{rec} = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT_e}\right)^{3/2} \int_o^\infty v^2 \exp\left\{-\frac{mv^2}{2kT_e}\right\} \nu_r(v)\, dv$$

7.5.2 Weakly Ionized Plasma

In the case of a weakly ionized plasma, the collisions between electrons are insignificant in the Boltzmann equation for f_o (since $\nu_e \ll \delta\nu$) and these colli-

sions may be disregarded in a first approximation. They are even less significant in the equation for f_1, since $\nu_e \ll \delta\nu \ll \nu$. Hence, the function f_1 in a homogeneous weakly ionized plasma (to terms of order δ) is given by the expression

$$f_1 = -\vec{u}\frac{\partial f_o}{\partial v}$$

where $\vec{u}$ satisfies the equation:

$$\frac{\partial \vec{u}}{\partial t} + \nu(u)\,\vec{u} = \frac{e\vec{E}}{m} + \frac{e}{mc}\,[\vec{u} \times \vec{H}_o]$$

$\vec{u}$ is the velocity of the directed motion of the electron. If this expression for f_1 is substituted into the general Boltzmann equation, the following equation for f_o is obtained:

$$\frac{\partial f_o}{\partial t} - \frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\left[\left(\delta\,(\nu_m^{el} + \nu_i)\frac{kT}{m} + \frac{2eEu}{3m}\right)\frac{\partial f_o}{\partial v} + \delta\, v(\nu_m^{el} + \nu_i) f_o\right]\right\} +$$

$$B_{ne}^{INELAS}\,(f_o) = 0$$

Depending on the relation between the time $1/\omega$, during which the electric field changes significantly, and the relaxation time for the function $f_o(\tau) \sim 1/\delta\nu$, we distinguish here cases of slowly varying field ($\omega \ll \delta\nu$, and rapid ones ($\omega \gg \delta\nu$) (the same as in the analysis of the electron temperature in elementary theory or in a strongly-ionized plasma). In the former case, which is quasi-stationary, the dependence of f_o on the time can be disregarded; in particular, this takes place naturally in the case of a constant electric field. On the other hand, for a rapidly alternating electric field, $\omega \gg \delta\nu$ the function f_o does not

have a chance to change as rapidly as the field; it therefore settles at a certain average level, independent of the time, and the variable deviations from this level are small, of amplitude on the order of $\delta\nu/\omega$ (the same as the observations of the electron temperature in elementary theory). Consequently, in both cases we can neglect in first approximation the term $\partial f_o/\partial t$, and thereby get rid in fact of the time variable. This allows us to find an analytical solution for the above equation for many important cases: for elastic collisions, in inert gases and in a molecular plasma. We now proceed to analyze these solutions.

7.5.2.1 Elastic Collisions

If all the collisions are elastic, $B_{ne}^{inel} = 0$. Therefore in a constant electric field E we have, $u = eE/m\nu$ at $H_o = 0$ and the above equation becomes:

$$\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\left[\left(\delta\,\nu\,\frac{kT}{m}+\frac{2e^2E^2}{3m^2\nu}\right)\frac{\partial f_o}{\partial v}+\delta\,\nu v f_o\right]\right\}=\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 J_v\right)=0$$

(Here $\nu = \nu(v) = \nu_m^{el} + \nu_i$). Multiplying this equation by v^2 and integrating from 0 to v, we see that $j_v = 0$, since in the absence of an electron source $[v^2 j_v]_{v=o} = 0$. Integrating now the equation $j_v = 0$ over the velocities, we obtain

$$f_o = C\exp\left\{-\int_0^v \frac{mv\,dv}{kT+\dfrac{2e^2E^2}{3m\delta\,\nu^2}}\right\}$$

We therefore obtain a Maxwellian distribution in a weak field, but in a strong field the distribution function f_o may differ substantially from Maxwellian, since

ν depends on v. For example, in a strong electric field upon collision with molecules - hard spheres - the function f_o is determined by the well known Druyvesteyn formula

$$f_o = c \exp\left\{-\frac{3m^2\delta}{8e^2E^2\ell^2}\right\} v^4$$

where $\ell = v/\nu(v) = 1/\pi a^2 N_m$ is the mean free path of the electron, C a constant determined from the normalization condition, and the term kT is neglected, this is permissible for a strong field.

The Druyvesteyn distribution at large electron velocities differs greatly from Maxwellian; it drops off much more rapidly than a Maxwellian one. The calculation of the function f_o with allowance for the exact dependence of the collision frequency and the velocity for different inert gases was made in references 7-10 and 7-11. The effect of a constant magnetic field is taken into account in reference 7-12 [the magnetic field changes the velocity $v(\nu)$, and accordingly f_o also changes].

We considered above only the case of a constant quasi-stationary electric field ($\nu \gg \delta\nu$) for in this case we can neglect in first approximation the derivative $\partial f_o/\partial t$. The function f_o now assumes the form

$$f_o = c \exp\left\{-\int_o^v \frac{mv\,dv}{kT + \left(\frac{e^2E_o^2}{3m\delta}\right)\varphi(v)}\right\}$$

Here the function $\varphi(v)$ without the magnetic field is equal to $[\nu^2 + \omega^2(v)]^{-1}$, and in the presence of a magnetic field

$$\varphi(\nu) = \frac{\cos^2\beta}{\omega^2 + \nu^2} + \frac{\sin^2\beta}{2[(\omega-\omega_H)^2 + \nu^2]} + \frac{\sin^2\beta}{2[(\omega-\omega_H)^2 + \nu^2]}$$

where β is the angle between $\mathcal{E}$ and H, ω_H is the gyromagnetic frequency, $\mathcal{E}_o$ is the amplitude and ω is the frequency of the alternating electric field. Such elastic collisions are produced in monatomic gases at low electron energies (up to 1 ev).

7.5.2.2 Inelastic Collisions in a Molecular Plasma

A molecular plasma may be defined as one formed by diatomic or polyatomic gases. In such a plasma, not only electronic levels, but also rotational and vibrational levels can be excited, the energy of such levels being exceedingly low ($\hbar\omega \approx 10^{-2}$ to 10^{-4} ev for rotational levels, and $\hbar\omega \approx 0.1$ to 0.5 ev for vibrational levels). Hence, inelastic collisions in such a plasma become important at electron energies of the order 10^{-2} ev (room temperature).

For a plasma composed of a diatomic gas such as hydrogen, oxygen, or nitrogen at electron energies less than 1 ev, the principal role is played by losses due to the excitation of vibrational levels. The energy of these vibrational levels is small compared with the average electron energy. For such cases, the integral for inelastic collisions for the function f_0 can be represented in the form:

$$B_{ne}^{inelas}(f_0) = -\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2 R_H(v)\left[\frac{kT}{m}\frac{\partial f_0}{\partial v} + v\, f_0\right]\right\}$$

where $R_H(v) = \sum_i r_{\omega_i}$ describes the energy losses of electrons in inelastic collisions. When this expression for B_{ne}^{inelas} is substituted into the Boltzmann equation for f_0 for a weakly ionized plasma ($\nu_e \ll \delta\nu$), it may be shown that the resulting equation coincides with the equation considered above for the case of elastic electron-neutral collisions. It is merely necessary to make the transformation:

$$\delta_{elas} = \frac{2m}{m} \rightarrow \delta_{inelas} = \frac{\delta_{el}(\nu^{el} + \nu_i) + R_H}{\nu}$$

For the case of a strong constant electric field, instead of a Druynesteyn distribution which one obtains for the case of elastic electron-neutral collisions, the zero order distribution function for a molecular plasma is given by:

$$f_0 = c \exp\left\{ -\frac{3\pi^2}{2e^2 E^2 \ell^2} \int_0^v v^2 \delta(v)\, dv \right\}$$

In order to obtain an explicit expression for f_0, it is necessary to calculate the function

$$\delta(v) = \frac{S_{el}(\nu^{el} + \nu_i) + R_H(v)}{\nu(v)}$$

This may be found if the function $R_H(v) = \sum_i r_{\omega_i}$ is known. However, to find r_{ω_i} it is necessary to know all the cross sections of the various inelastic processes.

Seaton[7-13] has calculated the cross sections for the excitation of electronic levels by electron impact for atomic oxygen and atomic nitrogen. Meyerott[7-14] has given the cross sections for excitation by electron impact of some of the levels of molecular nitrogen. The exchange of energy between electrons and the rotational motion of N_2 has been discussed by Gerjuoy and Stein[7-15]. The interaction of the electron with the quadrupole moment of the molecules was considered. Closed expressions for cross sections for an electron producing a charge of rotational quantum number $\Delta J = \pm 2$ as functions of J and of the ratio of initial and final electron momenta are given. If it is assumed that the average electron energy is much greater than kT_g (where T_g is the gas temperature), the fractional energy loss per collision may be regarded as small compared to unity for most electrons. Under this assumption and assuming N_J (the probability of the molecule being in rotational level J) is small when J is small, the cross section becomes:

$$\sigma_r = \frac{4\pi Q^2 a_0^2}{15}$$

where Q = molecular quadrupole moment in units of πa_0^2

Q = 0.96 for nitrogen

a_0 = first Bohr radius

Haas[7-16] and Schulz[7-17] have shown that there is a strong probability for electrons of 1 to 4 ev energy to lose energy equal to one or more vibrational quanta in N_2. These authors interpret the process as the formation of an unstable N_2^- ion which can decay into various vibrational levels of N_2. Haas' experiment was conducted with a swarm tube and demonstrated that the maximum of the cross section for this energy loss process occurs at 2.3 ev and is about 3×10^{-16} cm^2 in magnitude. There is some evidence that electrons also excite vibration in O_2 at energies of a few tenths of an electron volt. Because the investigation of this process by mobility and diffusion processes is complicated by attachment, it is difficult to assign cross sections to specific processes.

Since the cross sections for the various inelastic processes that may occur in a molecular plasma are not all well known, Ginzburg[7-7] has adopted another method to determine the total loss function

$$R(v) = \delta_{el}(\nu^{el} + \nu_i) + R_H(v) = \delta_{inel}\,\nu(v)$$

The fraction of the energy lost by the electron in a highly ionized molecular plasma (δ_{eff}) is related to the function R(v):

$$\int_0^\infty R(v)\, v^4 \exp\left[-\frac{mv^2}{2kT_e}\right] dv = \frac{3\sqrt{\pi}}{\sqrt{2}}\left(\frac{kT_e}{m}\right)^{3/2} \delta_{eff}(T_e)\,\nu_{eff}(T_e)$$

Here,

$$\delta_{eff} = \delta_{el} \frac{\nu_{eff}^{el}}{\nu_{eff}} + \frac{1}{\nu_{eff}(T_e - T)} \left[\frac{4\pi n}{3N_e k} \int_0^\infty v^4 B_{ne}^{inel} (f_{00})\, dv + (\nu_{ion} - \nu_{rec})\, T_e \right]$$

$$\nu_{eff} = \frac{\sqrt{2}}{3\sqrt{\pi}} \left(\frac{m}{kT_e} \right)^{3/2} \int_0^\infty \nu(v)\, v^4 \exp\left\{ - \frac{mv^2}{2kT_e} \right\} dv$$

The above relation may be considered as an integral equation for R(v), since the product $\nu_{eff}\, \delta_{eff}$ is known from experiment. Once R(n) is determined, $\delta(v) = R(v)/\nu(v)$ is known. The results of such a calculation for hydrogen, oxygen, nitrogen and air are given in Figure 7-1.

If the calculations for $\delta(v)$ are substituted into the expression

$$f_0 = c \exp \left\{ - \int_0^v \frac{mv\,dv}{kT + \left(\frac{e^2 E_0^2}{3m\delta} \right) \phi(v)} \right\}$$

the distribution function for electrons in a molecular plasma is determined.

The results of this calculation for electrons in hydrogen in a high frequency electric field are shown in Figure 7-2. The ordinates represent $-v^2/\bar{v}^2$ where $\bar{v}^2 = 2kT/m$ is the mean square electron velocity. The dotted line represents a Maxwellian distribution (the distribution would be Maxwellian if δ were independent of v, as it is for elastic collisions). It may be noted from Figure 7-2 that for electrons in H_2, the deviations of the distribution function from Maxwellian are not large.

Recently, Caldirola[7-18] has determined the velocity distribution function for electrons in a spatially homogeneous slightly ionized plasma under the action of a constant magnetic field (i.e., that of the Earth) and an alternating electric field

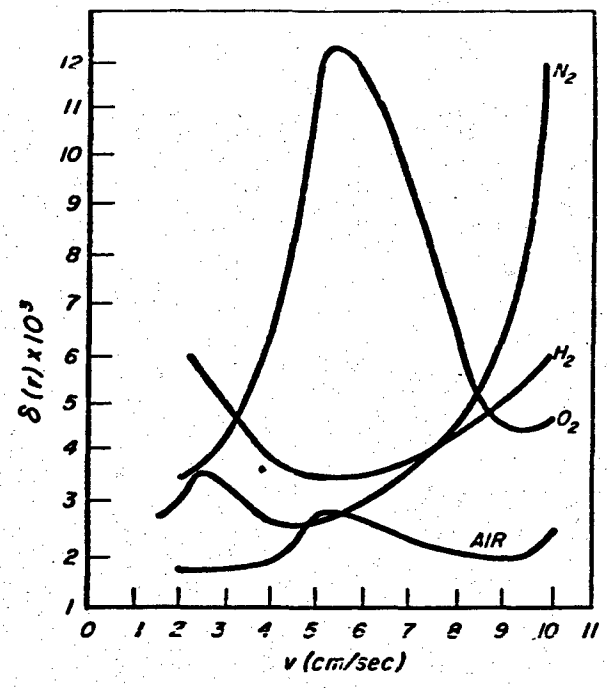

Figure 7-1

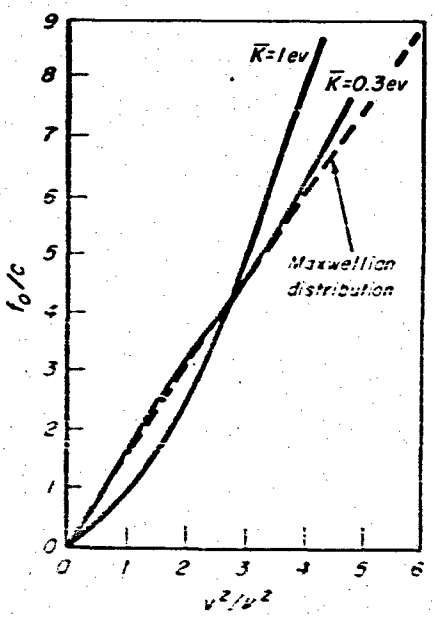

Figure 7-2

(the time varying magnetic contribution to the force exerted on an electron is assumed negligible compared with the electric one). Caldirola's expansion of the distribution function is similar to Ginzburg's[7-7] treatment, where an "effective" energy loss for electrons by collisions is written

$$\delta^{eff}(v) = \frac{R(v)}{\nu(v)}$$

Caldirola takes into account ionizing and recombination collisions under the assumption that steady state conditions pertain (which is accurate for the excitation of an airglow, but inapplicable for RF breakdown):

$$\int (B^{recom} + B^{ioniz}) \, d^3 \vec{v} = 0$$

Caldirola has given the following expression for the inelastic collision integral representing ionizing collisions:

$$B^{ion}_{en} = -\nu_i(v) \, f(v,t) + \eta \, \nu_i(v_i) \frac{v_i}{v} \, f(v_i,t) + \tilde{\eta} \, \nu_i(\tilde{v}_i) \frac{\tilde{v}_i}{v} f(\tilde{v}_i,t)$$

where $\nu_i(v) = N_A \, v \, \sigma_i$

N_A = number of molecules per cc

$v_i = (\eta v^2 + 2 \epsilon_i/m)^{1/2}$

$\tilde{v}_i = (\tilde{\eta} v^2 + 2 \epsilon_i/m)^{1/2}$

ϵ_i = ionization energy

η and $\tilde{\eta} = \eta/1-\eta$ are the parameters determing the energy distribution between the primary scattering and secondary electrons.

Carleton and Megill[7-19] have numerically computed solutions of the Boltzmann equation for the electron energy distribution in weakly ionized air. They have made the following assumptions:

(1) That the partially ionized plasma is situated in a static magnetic field.

(2) That an alternating electric field exists with electric vector perpendicular to the magnetic field.

(3) That the gas is homogeneous and the fields uniform in space.

(4) The degree of ionization is weak enough so that electron-electron and electron-ion collisions are negligible.

(5) The fields are such that the average electron energy is much larger than the thermal energies of the gas molecules (and that heating of the gas by the electrons is negligible).

(6) That ionization and recombination collisions are negligible.

(7) That electrons may lose energy in elastic collisions and may excite rotational, vibrational, and electronic degrees of freedom.

Carleton and Megill[7-19] describe the various elastic and inelastic energy loss processes by employing the known experimental cross sections. The gas was considered to be composed of N_2, O_2 and O in proportions corresponding to an altitude of about 100 km. The total electron velocity distribution function was expanded in the form

$$f(\vec{v},t) = f_0(v,t) + (\vec{V}\cdot\vec{E})\, f_1(v,t) + [\vec{v} \cdot (\vec{B} \times \vec{E})]\, f_2(v,t)$$

where f_1 and f_2 provide bulges on the distribution in the directions parallel to the electric field and to the Hall drift, respectively. Their final result for the electron distribution function exhibits a small hump on the calculated curve near the origin, which was due to the error introduced in the approximation which treats the

rotational cross section as a step function at 0.02 ev. It is claimed that the calculated curve is inaccurate beyond 4 or 5 ev because higher lying states have not been included. If such higher lying states had been included, the calculated curve would have remained below the Druyvesteyn distribution, instead of rising above it. It is definitively demonstrated that the resultant distribution differs greatly from the Maxwellian over the electron energy range 2 to 9 ev.

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8. Breakdown in a Spherical Cavity by a Remote RF Antenna

Of the several experiments on rf breakdown currently being performed, that employing an external rf source and a transparent spherical cavity most nearly approximates the geometry of the real case of breakdown in the atmosphere produced by a focussed ground antennae. Moreover, because of the lack of mode excitation (and the transparent enclosure), it is relatively simple to extend the scope of these experiments to include impurities and even solar radiation (according to our information on the solar flux at the various altitudes). For cavities large enough to avoid or inhibit the contributions of wall effects, the simulated solar flux will provide atomic species which are impossible to reproduce in most cavity work, but which are a fixture of the upper atmosphere.

It is to be emphasized that the results of the remote antenna breakdown experiments differ from those of comparable experiments employing microwave cavities. However, circumstances have dictated that no intensive effort be made to date to set up a detailed theory accounting for the observations. As a result of the special advantages of this particular configuration (a modulated conical geometry), we suggest a two-fold program.

First, the experimental work should be extended so as to include all combinations of pure gases, mixtures of gases, varying pressure, and simulated solar radiation.

Second, an attempt should be made to employ the Boltzmann Equation

to solve the time sequence of events leading to breakdown. Of course, this second part will be difficult, but for the first time, there will be available reliable numbers from an experiment with a simple and pertinent geometry.

9. A Direct Measurement of RF Breakdown in the Upper Atmosphere

It has been pointed out that there are two basic obstacles to the solution of the breakdown problem in the unbounded ambient atmospheric environment. These are:

1. The simplified phenomenological theory fails in the low pressure range even for pure gases.

2. The effects of an altering composition (this parameter is not well known above balloon heights), solar flux, etc. cannot be reliably included in either the simplified theory or to date in the more general Boltzmann approach. Consequently, it seems that a direct measurement upon the rf power required for breakdown be made in the atmosphere, say, in a parallel plate experiment.

Two vehicles, balloons and rockets, immediately suggest themselves, and each has its advantages and disadvantages. The two basic parameters of the vehicle are lifting capability (therefore altitude) and velocity with respect to the ambient atmosphere. Present balloon capabilities are limited to loads of 1000 lbs at 100,000 ft (30 km), whereas boosters are available to send 10 tons to such an altitude. On the other hand, large relative velocities between the carrier and the environment may lead to some complication in the interpretation of the experiment. Actually, it is anticipated that this effect be small particularly for pulsed breakdown.

Time does not permit an examination of the load required to provide rf fluxes of the order of 10^7 watts/m^2. But, it is strongly suggested here that a detailed and thorough feasibility study be made if it is considered that exact information on breakdown thresholds in the ambient atmosphere is required.

10. RF Breakdown in Perturbed Atmospheres

Once the problem of rf breakdown in the ambient upper atmosphere is solved, attention will be focussed upon the obvious sequel, rf breakdown in perturbed atmosphere. Here, there will be two general types of perturbations to be considered; namely, chemical seeding and nuclear detonations.

In regard to chemical seeding, there will again be two basically different chemicals to be considered. Particular chemicals have abnormally large (electron) attachment cross sections and therefore tend to inhibit breakdown. The halogens, hexafluorides, etc. belong to this class of substances. There will be a second class which can lower the breakdown threshold because of inherently lower ionization potentials. The alkali metals are good examples of these electron-givers.

The nuclear detonation - particularly at high altitudes - produces a radical perturbation of the environment over extensive volumes of the upper atmosphere. The basic difference between the chemical seeding and the nuclear detonation is that in the latter case the perturbation is produced by radiation rather than an injected impurity. Nevertheless, the detonation in addition to the dominant effect of an increased electron density (and therefore a lower breakdown field) can result in an alteration of the composition through such processes as dissociative recombination and subsequent atom-ion or atom-atom reactions.

No attempt is made here to more than delineate the general problem and suggest that some consideration be given to this area in the future.